$K \Leftrightarrow$
$(\exists z \in \bigcup \mathrm{Ob}\, \nu \forall K \in \mathrm{up}\, z \exists x \in \bigcup I : x \sqsubseteq K \wedge \exists z \in \bigcup \mathrm{Ob}\, \nu \forall K \in \mathrm{up}\, z \exists y \in \bigcup Y : y \sqsubseteq K) \vee$
$(\exists z \in \bigcup \mathrm{Ob}\, \nu \forall K \in \mathrm{up}\, z \exists x \in \bigcup J : x \sqsubseteq K \wedge \exists z \in \bigcup \mathrm{Ob}\, \nu \forall K \in \mathrm{up}\, z \exists y \in \bigcup Y : y \sqsubseteq K) \Leftrightarrow$
$(\exists z \in \bigcup \mathrm{Ob}\, \nu : (\forall K \in \mathrm{up}\, z \exists x \in \bigcup I : x \sqsubseteq K \wedge \forall K \in \mathrm{up}\, z \exists y \in \bigcup Y : y \sqsubseteq K)) \vee$
$(\exists z \in \bigcup \mathrm{Ob}\, \nu : (\forall K \in \mathrm{up}\, z \exists x \in \bigcup J : x \sqsubseteq K \wedge \forall K \in \mathrm{up}\, z \exists y \in \bigcup Y : y \sqsubseteq K)) \Leftrightarrow$
$(\exists z \in \bigcup \mathrm{Ob}\, \nu : (\forall K \in \mathrm{up}\, z : (\exists x \in \bigcup I : x \sqsubseteq K \wedge \exists y \in \bigcup Y : y \sqsubseteq K))) \vee$
$\exists z \in \bigcup \mathrm{Ob}\, \nu \forall K \in \mathrm{up}\, z : (\exists x \in \bigcup J : x \sqsubseteq K \wedge \exists y \in \bigcup Y : y \sqsubseteq K) \Leftrightarrow$
$I\ [\mathrm{SLA}(\nu)]^*\ Y \vee J\ [\mathrm{SLA}(\nu)]^*\ Y$.

To finish the proof we need to fulfill ?? in the above formula. For this it's enough to prove

$\forall K \in \mathrm{up}\, z \exists x \in \bigcup I \cup \bigcup J : x \sqsubseteq K \Rightarrow$
$\forall K \in \mathrm{up}\, z \exists x \in \bigcup I : x \sqsubseteq K \vee \forall K \in \mathrm{up}\, z \exists x \in \bigcup J : x \sqsubseteq K$.

If $z = \uparrow Z$ is a principal funcoid, then

$\forall K \in \mathrm{up}\, z \exists x \in \bigcup I \cup \bigcup J : x \sqsubseteq K \Rightarrow$
$\exists x \in \bigcup I \cup \bigcup J : x \sqsubseteq z \Rightarrow$
$\exists x \in \bigcup I : x \sqsubseteq z \vee \exists x \in \bigcup J : x \sqsubseteq z \Rightarrow$
$\forall K \in \mathrm{up}\, z \exists x \in \bigcup I : x \sqsubseteq K \vee \forall K \in \mathrm{up}\, z \exists x \in \bigcup J : x \sqsubseteq K$.

Following the idea of [[the proof in this math.stackexchange.com question|http://math.stackexchange.com/questions/562908/an-implication-involving-filters#562974]] it is easy to show that our implication is true if $\mathrm{up}\, z$ is closed regarding finite meets. See [[this page|Singularities funcoids: some special cases]] for attempts to set it true. The question is whether our statement holds for non-principal funcoids. Or is there a counterexampe?

3. Singularities funcoids: special cases proof attempts

To prove that $\mathrm{GR}(\Delta \times^{\mathsf{FCD}} \Delta)$ is closed under finite intersections, it's enough to prove that for every $f \in \mathrm{GR}(\Delta \times^{\mathsf{FCD}} \Delta)$ there is a positive ε such that $\forall x \in]-\varepsilon; \varepsilon[: fx \in \Delta$.

Really, under this assumption:

For $g \in \mathrm{GR}(\Delta \times^{\mathsf{FCD}} \Delta)$ exists $\zeta > 0$ such that $\forall x \in]-\zeta; \zeta[: gx \in \Delta$. Let $\eta = \min\{\varepsilon, \zeta\}$. So $\forall x \in]-\eta; \eta[: (\langle f \rangle x \in \Delta \wedge \langle g \rangle x \in \Delta)$ and so $\forall x \in]-\eta; \eta[: \langle f \cap g \rangle x \in \Delta$ that is $\forall x \in]-\eta; \eta[: \langle \uparrow^{\mathsf{FCD}} (f \cap g) \rangle^* \{x\} \sqsupseteq \Delta$ and consequently $f \cap g \in \mathrm{GR}(\Delta \times^{\mathsf{FCD}} \Delta)$.