CHAPTER 13

# Singularities

**Very** rough draft.

## 1. Singularities funcoids: some special cases

We attempt to prove that $\operatorname{up} z$ is closed regarding finite intersections.

For consideration of this, let's consider two special cases (first of which is a specialization of the second).

Let $\mu = \nu$ be the natural proximity on real numbers $\mathbb{R}$.

Let $\Delta$ is the entourage filter of zero.

1. $z = \Delta \times^{\mathsf{FCD}} \Delta$.
2. $z = \nu \circ (\uparrow^{\mathsf{FCD}} f)|_\Delta$ for an arbitrary function $f : \mathbb{R} \to \mathbb{R}$.

(1) is [[also formulated in elementary terms|http://math.stackexchange.com/questions/568513/is-a-set-closed-under-finite-intersections-about-filters]] (without using funcoids).

These two above conjectures are shown to be false by a counter-example in [[this blog post|http://portonmath.wordpress.com/2013/12/18/a-negative-result-on-a-conjecture/]]. It is a discouraging result as it seems from it the plain funcoids can't be used for the multilevel theory of singularities.

## 2. Using plain funcoids

This way if we succeed is the best way to create metasingular numbers because, it (if we succeed) involves just funcoids not some fancy generalization of funcoids.

Approximate definition of "singularity level": //Singularity level// is a transitive, $T_2$-separable endofuncoid.

Now define the funcoid $\nu_{i+1} = \operatorname{SLA}(\nu_i)$:

$\operatorname{Ob}(\nu_{i+1})$ is defined as the set of all generalized limits (having fixed $\mu$, $\nu$, and $G$).

$X\ [\nu_{i+1}]^*\ Y \Leftrightarrow \exists z \in \bigcup \operatorname{Ob} \nu \forall K \in \operatorname{up} z \exists x \in \bigcup X, y \in \bigcup Y : x, y \sqsubseteq K$.

The trouble is to prove that the funcoid $\nu_{i+1}$ exists (is really a funcoid).

$\neg(X\ [\nu_{i+1}]^*\ \emptyset)$ and $\neg(\emptyset\ [\nu_{i+1}]^*\ Y)$ are obvious. We need to prove

$$I \cup J\ [\nu_{i+1}]^*\ Y \Leftrightarrow I\ [\nu_{i+1}]^*\ Y \vee J\ [\nu_{i+1}]^*\ Y$$

and

$$X\ [\nu_{i+1}]^*\ I \cup J \Leftrightarrow X\ [\nu_{i+1}]^*\ I \vee X\ [\nu_{i+1}]^*\ J.$$

Let's attempt to prove the first of the above equations (the second is dual).

$I \cup J\ [\operatorname{SLA}(\nu)]^*\ Y \Leftrightarrow$

$\exists z \in \bigcup \operatorname{Ob} \nu \forall K \in \operatorname{up} z \exists x \in \bigcup I \cup \bigcup J, y \in \bigcup Y : x, y \sqsubseteq K \Leftrightarrow$

$\exists z \in \bigcup \operatorname{Ob} \nu \forall K \in \operatorname{up} z : (\exists x \in \bigcup I \cup \bigcup J : x \sqsubseteq K \wedge \exists y \in \bigcup Y : y \sqsubseteq K) \Leftrightarrow$

$\exists z \in \bigcup \operatorname{Ob} \nu \forall K \in \operatorname{up} z \exists x \in \bigcup I \cup \bigcup J : x \sqsubseteq K \wedge$

$\exists z \in \bigcup \operatorname{Ob} \nu \forall K \in \operatorname{up} z \exists y \in \bigcup Y : y \sqsubseteq K \Leftrightarrow$

??

$\exists z \in \bigcup \operatorname{Ob} \nu : (\forall K \in \operatorname{up} z \exists x \in \bigcup I : x \sqsubseteq K \vee$

$\forall K \in \operatorname{up} z \exists x \in \bigcup J : x \sqsubseteq K) \wedge \exists z \in \bigcup \operatorname{Ob} \nu \forall K \in \operatorname{up} z \exists y \in \bigcup Y : y \sqsubseteq K \Leftrightarrow$

$(\exists z \in \bigcup \operatorname{Ob} \nu \forall K \in \operatorname{up} z \exists x \in \bigcup I : x \sqsubseteq K \vee$

$\exists z \in \bigcup \operatorname{Ob} \nu \forall K \in \operatorname{up} z \exists x \in \bigcup J : x \sqsubseteq K) \wedge \exists z \in \bigcup \operatorname{Ob} \nu \forall K \in \operatorname{up} z \exists y \in \bigcup Y : y \sqsubseteq$