PROOF. $\langle f\rangle^* A \sqcap \langle f\rangle^*((\operatorname{Src} f) \setminus A) = 0^{\operatorname{Dst} f}$ because f is monovalued.
$\langle f\rangle^* A \sqcup \langle f\rangle^*((\operatorname{Src} f) \setminus A) = 1^{\operatorname{Dst} f}$.
Therefore $\langle f\rangle^* A$ is a principal filter (theorem 49 in [4]). So f is co-complete.
That f is complete follows from symmetry. □

For wider class of pointfree funcoids the concept of bijection does not make sense. Instead we would want a structure preserving map to be *order isomorphism.*

Actually, for mapping between $\mathscr{P}A$ and $\mathscr{P}B$ where A and B are some sets (including the above considered case of funcoids from A to B) bijection and order isomorphism are essentially the same:

PROPOSITION 2076. Bijections F between sets A and B bijectively correspond to order isomorphisms f between $\mathscr{P}A$ and $\mathscr{P}B$ by the formula $f = \langle F\rangle$.

PROOF. Let F is a bijection. Then $X \subseteq Y \Rightarrow \langle F\rangle X \subseteq \langle F\rangle Y$ and $\langle F^{-1}\rangle\langle F\rangle X = X$ for every sets $X, Y \in \mathscr{P}A$. Thus $f = \langle F\rangle$ is an order isomorphism.

Let now f is an order isomorphism between $\mathscr{P}A$ and $\mathscr{P}B$. Then $f(\{x\})$ is a singleton for every $x \in A$. Take $F(x)$ to the unique y such that $f(\{x\}) = \{y\}$. Obviously f is a bijection and $f = \langle F\rangle$. □

For arbitrary pointfree funcoids isomorphisms do not necessarily preserve structure. It holds only for *increasing pointfree funcoids*:

DEFINITION 2077. I call a pointfree funcoid f *increasing* iff $\langle f\rangle$ and $\langle f^{-1}\rangle$ are monotone functions.

PROPOSITION 2078. If f is an increasing isomorphism of the category of pointfree funcoids then $\langle f\rangle$ is an order isomorphism.

PROOF. We have: $\langle f\rangle \circ \langle f^{-1}\rangle = \langle f \circ f^{-1}\rangle = \langle \mathrm{id}^{\mathsf{FCD}}_{\mathfrak{B}}\rangle = \mathrm{id}_{\mathfrak{B}}$ and $\langle f^{-1}\rangle \circ \langle f\rangle = \langle f^{-1} \circ f\rangle = \langle \mathrm{id}^{\mathsf{FCD}}_{\mathfrak{A}}\rangle = \mathrm{id}_{\mathfrak{A}}$. Thus $\langle f\rangle$ is a bijection.
$\langle f\rangle$ is increasing and bijective. □

REMARK 2079. Non-increasing isomorphisms of the category of pointfree funcoids are against sound mind, they don't preserve the structure of the source, that is for them $\langle f\rangle$ or $\langle f^{-1}\rangle$ are not order isomorphisms.

OBVIOUS 2080. Isomorphisms of cont(**Pos**) and cont(**mePos**) are order isomorphisms.

6. Direct products

FiXme: Now this section is a complete mess. Clean it up.

Consider the category **contFcd** which is the full subcategory cont(**mePos**) restricted to objects which are essentially increasing pointfree funcoids.

Let $f_1 : Y \to X_1$ and $f_2 : Y \to X_2$ are morphisms of **contFcd**.

The product object is $X_1 \times^{(C)} X_2$ (cross composition product of funcoids used). It is easy to see that $X_1 \times^{(C)} X_2$ is an object of **contFcd** that is an endo-funcoid.

The morphism $f_1 \times^{(D)} f_2 : Y \to X_1 \times^{(C)} X_2$ is defined by the formula $(f_1 \times^{(D)} f_2)y = f_1 y \times^{\mathsf{FCD}} f_2 y$.

$f_1 \times^{(D)} f_2$ is monovalued and entirely defined because so are f_1 and f_2.

$$(f_1 \times^{(D2)} f_2)y = \bigcup\left\{ f_1 Y \times^{\mathsf{FCD}} f_2 Y \quad\middle|\quad Y \in \operatorname{atoms}^{\mathfrak{A}} y \right\}.$$

FiXme: Is $(f_1 \times^{(D2)} f_2)$ a pointfree funcoid?

To prove that it is really a morphism we need to show

$$(f_1 \times^{(D)} f_2) \circ Y \sqsubseteq (X_1 \times^{(C)} X_2) \circ (f_1 \times^{(D)} f_2)$$