CHAPTER 8

Funcoids as closed sets

Idea [6] by Todd Trimble.

FiXme: https://ncatlab.org/toddtrimble/published/topogeny and https://math.stackexchange.com/q/2681502/4876

FiXme: What about the infinite products?

Theorem 2061. The set of staroids $\mathscr{P}X_1 \times \cdots \times \mathscr{P}X_n \to 2$ is order isomorphic to co-frame of closed subsets of topological product $\beta X_1 \times \cdots \times \beta X_n$.

Proof. $\mathscr{P}X_1 \times \cdots \times \mathscr{P}X_n \to 2$ can be order-embedded to the frame of ideals $\mathfrak{J}(\mathscr{P}X_1 \times \cdots \times \mathscr{P}X_n)$ what is dual (check!) to the frame of ideals of the distributive lattice $\mathscr{P}X_1 \otimes \cdots \otimes \mathscr{P}X_n$. This by ?? is the coproduct $\sum_i \mathscr{P}X_i$ in the category of boolean algebras. By Stone duality it is isomorphic to the topology of it spectrum $\beta X_1 \times \cdots \times \beta X_n$. □

Elements of $\beta X_1 \times \cdots \times \beta X_n$ are closed subsets. So every n-staroid one-to-one corresponds to a closed set of $\beta X_1 \times \cdots \times \beta X_n$.

Note that $\beta X_1 \times \cdots \times \beta X_n$ is a compact Hausdorff space (as a product of compact Hausdorff spaces).

It seems that there is an easy way to describe the above order embedding in both directions:

$$f \mapsto \left\{ \frac{(x_1, \ldots, x_n)}{x_1, \ldots, x_n \in \operatorname{atoms}^{\mathscr{F}}, x_1 \times^{\mathsf{FCD}} \cdots \times^{\mathsf{FCD}} x_n \sqsubseteq f} \right\};$$

$$X \mapsto \bigsqcup \left\{ \frac{p_1 \times^{\mathsf{FCD}} \cdots \times^{\mathsf{FCD}} p_n}{p \in X} \right\}.$$

FiXme: Try to prove that composition of funcoids is isomorphic to composition of relations $\beta A \times \beta B$.