CHAPTER 7

Mappings between endofuncoids and topological spaces

Oreder topologies reversely to set-theoretic inclusion. That is for topologies t and s we set $t \sqsubseteq s \Leftrightarrow t \supseteq s$. (Intuitively: The less is the topology, the lesser are its open sets.)

Let's study mappings between topological spaces and endofuncoids.

Definition 2049. Let t be a topology.

1°. $F^\star t = \bigsqcup_{x \in \operatorname{Ob} t} \left(\{x\} \times \prod^{\mathscr{F}} \left\{ \frac{E \in t}{x \in E} \right\} \right)$;

2°. $(F_\star t)E = \bigcap \left\{ \frac{D \in t}{E \subseteq D} \right\}$.

Proposition 2050. Let t be a topology.

1°. $F^\star t$ is complete, reflexive, transitive funcoid.

2°. $F_\star t$ is co-complete, reflexive, transitive funcoid.

3°. $F^\star$ and $F_\star$ are injections.

4°. $F_\star t = (F^\star t)^{-1}$.

Proof. By theorem 785. □

Definition 2051. Let f be an endofuncoid.

$$T f = \left\{ \frac{E \in \mathscr{P} \operatorname{Ob} f}{\forall x \in E : \langle f \rangle \{x\} \sqsubseteq E} \right\}.$$

Proposition 2052. $T f$ is a topology.

Proof. Union of open sets is open. $S \subseteq T f \Rightarrow \forall E \in S \forall x \in E : \langle f \rangle x \sqsubseteq E \Rightarrow \forall x \in \bigcup S : \langle f \rangle x \sqsubseteq \bigcup S$

Intersection of two open sets is open. Let $X, Y \in T f$. Then $\forall x \in X : \langle f \rangle x \sqsubseteq X$ and $\forall x \in Y : \langle f \rangle x \sqsubseteq Y$. So if $x \in X \cap Y$ then $\langle f \rangle x \sqsubseteq X$ and $\langle f \rangle x \sqsubseteq Y$, so $\langle f \rangle x \sqsubseteq X \cap Y$. So $X \cap Y \in T f$.

$\operatorname{Ob} f$ is an open set. Obvious.

□

Obvious 2053. $T f = \left\{ \frac{E \in \mathscr{P} \operatorname{Ob} f}{\langle \operatorname{Compl} f \rangle E \sqsubseteq E} \right\}$.

In some reason when starting this research I assumed that the following funcoid (for every endofuncoid f) is a Kuratowski closure:

$$1 \sqcup \operatorname{CoCompl} f \sqcup (\operatorname{CoCompl} f)^2 \sqcup \ldots.$$

It is not true:

Example 2054. There exists such a co-complete endofuncoid f that $1 \sqcup f \sqcup f^2 \sqcup \ldots$ is not transitive that is

$$(1 \sqcup f \sqcup f^2 \sqcup \ldots) \circ (1 \sqcup f \sqcup f^2 \sqcup \ldots) \neq 1 \sqcup f \sqcup f^2 \sqcup \ldots$$