We need to check that p is a **Filt**-morphism that is $p(\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}) \sqsubseteq \mathcal{A}$ what is obvious.

Similarly for the right projection q.

It remains to check the universal property: Let $\mathcal{C}$ be a filter and $f : \mathcal{C} \to \mathcal{A}$, $g : \mathcal{C} \to \mathcal{B}$. We need to prove that there are a unique $u : \mathcal{C} \to \mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}$ such that $f = p \circ u$ and $g = q \circ u$. Denote $h(z) = (f(z), g(z))$.

h is the unique function $\operatorname{Base}(\mathcal{C}) \to \operatorname{Base}(\mathcal{A}) \times \operatorname{Base}(\mathcal{B})$ such that $f = p \circ h$ and $g = q \circ h$, so it remains to check that h is a morphism of **Filt** that is $\langle h \rangle \mathcal{C} \sqsubseteq \mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}$, what obviously follows from $\langle f \rangle \mathcal{C} \sqsubseteq \mathcal{A}$ and $\langle g \rangle \mathcal{C} \sqsubseteq \mathcal{B}$. □

THEOREM 2035. $\prod^{\mathsf{RLD}*}$ together with projections Pr_k is a categorical product in **Filt**.

PROOF. Consider an indexed family $\mathcal{A}$ of objects.

Denote p_k the k-th projection from $\prod_{i \in \operatorname{dom} \mathcal{A}} \operatorname{Base}(\mathcal{A}_i)$.

We need to check that p_k s a **Filt**-morphism that is $p_k \left(\prod^{\mathsf{RLD}*} \mathcal{A} \right) \sqsubseteq \mathcal{A}_k$ what is obvious.

It remains to check the universal property: Let $\mathcal{C}$ be a filter and $f_k : \mathcal{C} \to \mathcal{A}_k$. We need to prove that there are a unique $u : \mathcal{C} \to \prod^{\mathsf{RLD}*} \mathcal{A}$ such that $f_k = p_k \circ u$. Denote $h(z) = \lambda i \in \operatorname{dom} \mathcal{A} : f_i z$.

h is the unique function $\operatorname{Base}(\mathcal{C}) \to \prod_{i \in \operatorname{dom} \mathcal{A}} \operatorname{Base}(\mathcal{A}_i)$ such that $f_k = p_k \circ h$, so it remains to check that h is a morphism of **Filt** that is $\langle h \rangle \mathcal{C} \sqsubseteq \prod^{\mathsf{RLD}*} \mathcal{A}$. It follows from

$$\operatorname*{Pr}_i^{\mathsf{RLD}} \langle h \rangle \mathcal{C} = \bigsqcap \langle Pr_i \rangle^* \langle h \rangle^* \operatorname{up} \mathcal{C} = \bigsqcap \langle \operatorname*{Pr}_i \circ h \rangle^* \operatorname{up} \mathcal{C} = \bigsqcap \langle f_i \rangle^* \operatorname{up} \mathcal{C} = \langle f_i \rangle \mathcal{C} \sqsubseteq \mathcal{A}_i.$$

□