DEFINITION 2021. $f/\sim = \uparrow\pi\circ f\circ\uparrow\pi^{-1}$ for every morphism f.

OBVIOUS 2022. $\operatorname{Ob}(f/\sim) = (\operatorname{Ob} f)/r$.

OBVIOUS 2023. $f/\sim = \langle\uparrow^{\mathsf{FCD}}\pi\times^{(C)}\uparrow^{\mathsf{FCD}}\pi\rangle f$ for every morphism f.

To define co-equalizers of morphisms f and g let $\sim$ be is the smallest equivalence relation such that $fx = gx$.

LEMMA 2024. Let $f : X \to Y$ be a morphism of the category $\operatorname{cont}(\mathcal{C})$ where $\mathcal{C}$ is a concrete category (so $Wf = \uparrow\varphi$ for a **Rel**-morphism φ because f is principal) such that φ respects $\sim$. Factor it $\varphi = u\circ\pi$ where $u : \operatorname{Ob}(X/\sim) \to \operatorname{Ob} Y$ using properties of **Set**. Then u is a morphism of $\operatorname{cont}(\mathcal{C})$ (that is a continuous function $X/\sim\to Y$).

PROOF. $f\circ X\circ f^{-1}\sqsubseteq Y$; $\uparrow u\circ\uparrow\pi\circ X\circ\uparrow\pi^{-1}\circ\uparrow u^{-1}\sqsubseteq Y$; $\uparrow u\in\mathrm{C}(\uparrow\pi\circ X\circ\uparrow\pi^{-1}, Y) = \mathrm{C}(X/\sim, Y)$. □

THEOREM 2025. The following is a co-equalizer of parallel morphisms $f, g : A \to B$ of category $\operatorname{cont}(\mathcal{C})$:
- the object $Y = f/\sim$;
- the morphism π considered as a morphism $B \to Y$.

PROOF. Let $z\circ f = z\circ g$ for some morphism z.

Let's prove $u\circ\pi = z$ for some $u : Y \to \operatorname{Dst} z$. Really, as a morphism of **Set** it exists and is unique.

$\operatorname{Src} z\in Y$. Thus $z = u\circ\pi$ for some u (by properties of **Set**). The function u is a $\operatorname{cont}(\mathcal{C})$-morphism because of the lemma above. It is unique by properties of **Set** (π obviously respects equivalence classes). □

3. Rest

THEOREM 2026. The categories $\operatorname{cont}(\mathcal{C})$ (for example in **Fcd** and **Rld**) are complete. FiXme: Note that small complete category is a preorder!

PROOF. They have products and equalizers. □

THEOREM 2027. The categories $\operatorname{cont}(\mathcal{C})$ (for example in **Fcd** and **Rld**) are co-complete.

PROOF. They have co-products and co-equalizers. □

DEFINITION 2028. I call morphisms f and g of a category with embeddings *equivalent* ($f\sim g$) when there exist a morphism p such that $\operatorname{Src} p\sqsubseteq\operatorname{Src} f$, $\operatorname{Src} p\sqsubseteq\operatorname{Src} g$, $\operatorname{Dst} p\sqsubseteq\operatorname{Dst} f$, $\operatorname{Dst} p\sqsubseteq\operatorname{Dst} g$ and $\iota_{\operatorname{Src} f,\operatorname{Dst} f}p = f$ and $\iota_{\operatorname{Src} g,\operatorname{Dst} g}p = g$.

PROBLEM 2029. Find under which conditions:

1°. Equivalence of morphisms is an equivalence relation.
2°. Equivalence of morphisms is a congruence for our category.