PROOF. $a \left[\prod^{(A)} F\right] b \Leftrightarrow \forall i \in \operatorname{dom} F : \Pr^{\mathsf{RLD}}_i a \; [F_i] \; \Pr^{\mathsf{RLD}}_i b \Leftrightarrow$ $\forall i \in \operatorname{dom} F : \left\langle \left(\pi_i^{\mathsf{FCD}\left(\prod_{i\in n} \operatorname{Dst} F\right)}\right)^{-1} \right\rangle [F_i] \left\langle \pi_i^{\mathsf{FCD}\left(\prod_{i\in n} \operatorname{Src} F\right)} \right\rangle \Leftrightarrow$ $\forall i \in \operatorname{dom} F : a \left[\left(\pi_i^{\mathsf{FCD}\left(\prod_{i\in n} \operatorname{Dst} F\right)}\right)^{-1} \circ F_i \circ \pi_i^{\mathsf{FCD}\left(\prod_{i\in n} \operatorname{Src} F\right)}\right] b \Leftrightarrow$ $a \left[\prod_{i\in n} \left(\left(\pi_i^{\mathsf{FCD}\left(\prod_{i\in n} \operatorname{Dst} F\right)}\right)^{-1} \circ F_i \circ \pi_i^{\mathsf{FCD}\left(\prod_{i\in n} \operatorname{Src} F\right)}\right)\right] b$ for ultrafilters a and b. □

COROLLARY 2016. $\prod^{(L)} F = \prod^{(A)} F$ is F is a small indexed family of funcoids.

7. Further plans

Does the formula $\prod^{(L)}_{i\in n}(g_i \circ f_i) = \prod^{(L)} g \circ \prod^{(L)} f$ hold?
Coordinate-wise continuity.

8. Cartesian closedness

We are not only to prove (or maybe disprove) that our categories are cartesian closed, but also to find (if any) explicit formulas for exponential transpose and evaluation.

"Definition" A category is //cartesian closed// iff:

1°. It has finite products.
2°. For each objects A, B is given an object $\operatorname{MOR}(A, B)$ (//exponentiation//) and a morphism $\varepsilon^{\mathbf{Dig}}_{A,B} : \operatorname{MOR}(A, B) \times A \to B$.
3°. For each morphism $f : Z \times A \to B$ there is given a morphism (//exponential transpose//) $\sim f : Z \to \operatorname{MOR}(A, B)$.
4°. $\varepsilon_{B,C} \circ (\sim f \times 1_A) = f$ for $f : A \to B \times C$.
5°. $\sim (\varepsilon_{B,C} \circ (g \times 1_A)) = g$ for $g : A \to \operatorname{MOR}(B, C)$.

We will also denote $f \mapsto (-f)$ the reverse of the bijection $f \mapsto (\sim f)$.

Our purpose is to prove (or disprove) that categories **Dig**, **Fcd**, and **Rld** are cartesian closed. Note that they have finite (and even infinite) products is already proved.

Alternative way to prove: you can prove that the functor $- \times B$ is left adjoint to the exponentiation $-^B$ where the counit is given by the evaluation map.

8.1. Definitions. Categories **Dig**, **Fcd**, and **Rld** are respectively categories of:

1°. discretely continuous maps between digraphs;
2°. (proximally) continuous maps between endofuncoids;
3°. (uniformly) continuous maps between endoreloids.

"Definition" //Digraph// is an endomorphism of the category **Rel**.

For a digraph A we denote $\operatorname{Ob} A$ the set of vertexes or A and $\operatorname{GR} A$ the set of edges or A.

"Definition" Category **Dig** of digraphs is the category whose objects are digraphs and morphisms are discretely continuous maps between digraphs. That is morphisms from a digraph μ to a digraph ν are functions (or more precisely morphisms of **Set**) f such that $f \circ \mu \sqsubseteq \nu \circ f$ (or equivalently $\mu \sqsubseteq f^{-1} \circ \nu \circ f$ or equivalently $f \circ \mu \circ f^{-1} \sqsubseteq \nu$).

"Remark" Category of digraphs is sometimes defined in an other (non equivalent) way, allowing multiple edges between two given vertices.