4.2. Reloids.

Definition 2010. $\mathbf{Rld} \overset{\text{def}}{=} \operatorname{cont} \mathsf{RLD}$.

Let F be a family of endoreloids.

The cartesian product $\prod^{(Q)} X \overset{\text{def}}{=} \prod X$.

I define $\pi_i = \pi_i^X \in \mathsf{RLD}(\prod X, X_i)$ as the principal reloid corresponding to the i-th projection. (Here π is entirely defined.)

The disjoint union $\coprod^{(Q)} X \overset{\text{def}}{=} \coprod X$.

I define $\iota_i = \iota^X \in \mathsf{RLD}(X_i, \coprod X)$ as the principal reloid corresponding to the i-th canonical injection. (Here ι is entirely defined.)

Let $\bigotimes$ and $\bigoplus$ are defined in the same way as in category **Set**.

Obvious 2011. $\pi_i \circ \bigotimes f = f_i$; $\bigotimes_{i\in n}(\pi_i \circ f) = f$.

Obvious 2012. $(\bigoplus f) \circ \iota_i = f_i$; $\bigoplus_{i\in n}(f \circ \iota_i) = f$.

It is easy to show that π_i is entirely defined monovalued, and ι_i is metacomplete and co-metacomplete.

Thus we are under conditions for both canonical products and canonical co-products and thus both $\prod^{(L)} F$ and $\coprod^{(L)} F$ are defined.

It is trivial that for uniform spaces infimum product of reloids coincides with product uniformilty.

5. Initial and terminal objects

Initial object of **Fcd** is the endofuncoid $\uparrow^{\mathsf{FCD}(\emptyset,\emptyset)} \emptyset$. It is initial because it has precisely one morphism o (the empty set considered as a function) to any object Y. o is a morphism because $o \circ \uparrow^{\mathsf{FCD}(\emptyset,\emptyset)} \emptyset \sqsubseteq Y \circ o$.

Proposition 2013. Terminal objects of **Fcd** are exactly $\uparrow^{\mathscr{F}} \{*\} \times^{\mathsf{FCD}} \uparrow^{\mathscr{F}} \{*\} = \uparrow^{\mathsf{FCD}} \{(*,*)\}$ where $*$ is an arbitrary point.

Proof. In order for a function $f : X \rightarrow \uparrow^{\mathsf{FCD}} \{(*,*)\}$ be a morphism, it is required exactly $f \circ X \sqsubseteq \uparrow^{\mathsf{FCD}} \{(*,*)\} \circ f$

$f \circ X \sqsubseteq (f^{-1} \circ \uparrow^{\mathsf{FCD}} \{(*,*)\})^{-1}$; $f \circ X \sqsubseteq (\{*\} \times^{\mathsf{FCD}} \langle f^{-1}\rangle\{*\})^{-1}$; $f \circ X \sqsubseteq \langle f^{-1}\rangle\{*\} \times^{\mathsf{FCD}} \{*\}$ what true exactly when f is a constant function with the value $*$. □

If $n = \emptyset$ then $Z = \{\emptyset\}$; $\prod^{(L)} \emptyset = \max \mathsf{FCD}(Z,Z) = \uparrow^{\mathscr{F}} \{\emptyset\} \times^{\mathsf{FCD}} \uparrow^{\mathscr{F}} \{\emptyset\} = \uparrow^{\mathsf{FCD}} \{(\emptyset,\emptyset)\}$.

FiXme: Initial and terminal objects of **Rld**.

6. Canonical product and subatomic product

FiXme: Confusion between filters on products and multireloids.

Proposition 2014. $\Pr_i^{\mathsf{RLD}}|_{\mathfrak{F}(Z)} = \langle \pi_i \rangle$ for every index i of a cartesian product Z.

Proof. If $\mathcal{X} \in \mathfrak{F}(Z)$ then $(\Pr_i^{\mathsf{RLD}}|_{\mathfrak{F}(Z)})\mathcal{X} = \Pr_i^{\mathsf{RLD}} \mathcal{X} = \prod^{\mathscr{F}} \langle \Pr_i \rangle^* \mathcal{X} = \prod \langle \pi_i \rangle \operatorname{up} \mathcal{X} = \langle \pi_i \rangle \mathcal{X}$. □

Proposition 2015. $\prod^{(A)} F = \bigsqcap_{i\in n} \left(\left(\pi_i^{\mathsf{FCD}\left(\prod_{i\in n} \operatorname{Dst} F\right)} \right)^{-1} \circ F_i \circ \pi_i^{\mathsf{FCD}\left(\prod_{i\in n} \operatorname{Src} F\right)} \right)$.