REMARK 1973. In some important examples the function π is entire, that is $\operatorname{dom}\pi$ is the set of all small indexed families of objects of $\mathcal{C}$. However there are also some important examples where it is partial.

DEFINITION 1974. *Infimum product* $\prod F$ (such that π is defined at $\lambda j\in n : \operatorname{Src} F_j$ and $\lambda j\in n : \operatorname{Dst} F_j$) is defined by the formula

$$\prod^{(L)} F = \bigsqcap_{i\in\operatorname{dom} F} ((\pi_i^{\lambda j\in n:\operatorname{Dst} F_j})^\dagger \circ F_i \circ \pi_i^{\lambda j\in n:\operatorname{Src} F_j}).$$

This formula can be (over)simplified to:

$$\prod^{(L)} F = \bigsqcap_{i\in\operatorname{dom} F} ((\pi_i^{\operatorname{Dst}\circ F})^\dagger \circ F_i \circ \pi_i^{\operatorname{Src}\circ F}).$$

REMARK 1975. $(\pi_i^{\lambda j\in n:\operatorname{Dst} F_j})^\dagger \circ F_i \circ \pi_i^{\lambda j\in n:\operatorname{Src} F_j} \in \operatorname{Hom}\left(\prod^{(Q)}_{j\in n}\operatorname{Src} F_j, \prod^{(Q)}_{j\in n}\operatorname{Dst} F_j\right)$ are properly defined and have the same sources and destination (whenever $i\in\operatorname{dom} F$ is), thus the meet in the formulas is properly defined.

REMARK 1976. Thus

$$F_0\times^{(L)} F_1 = ((\pi_0^{(\operatorname{Dst} F_0,\operatorname{Dst} F_1)})^\dagger\circ F_0\circ\pi_0^{(\operatorname{Src} F_0,\operatorname{Src} F_1)})\sqcap((\pi_1^{(\operatorname{Dst} F_0,\operatorname{Dst} F_1)})^\dagger\circ F_1\circ\pi_1^{(\operatorname{Src} F_0,\operatorname{Src} F_1)})$$

that is product is defined by a pure algebraic formula.

PROPOSITION 1977. $\prod^{(L)} F = \max\left\{\frac{\Phi\in\operatorname{Hom}\left(\prod^{(Q)}_{j\in n}\operatorname{Src} F_j,\prod^{(Q)}_{j\in n}\operatorname{Dst} F_j\right)}{\forall i\in n:\Phi\sqsubseteq(\pi_i^{\lambda j\in n:\operatorname{Dst} F_j})^\dagger\circ F_i\circ\pi_i^{\lambda j\in n:\operatorname{Src} F_j}}\right\}$.

PROOF. By definition of meet on a complete lattice. □

COROLLARY 1978. $\prod^{(L)} F = \bigsqcup\left\{\frac{\Phi\in\operatorname{Hom}\left(\prod^{(Q)}_{j\in n}\operatorname{Src} F_j,\prod^{(Q)}_{j\in n}\operatorname{Dst} F_j\right)}{\forall i\in n:\Phi\sqsubseteq(\pi_i^{\lambda j\in n:\operatorname{Dst} F_j})^\dagger\circ F_i\circ\pi_i^{\lambda j\in n:\operatorname{Src} F_j}}\right\}$.

THEOREM 1979. Let π_i^X be metamonovalued morphisms. If $S \in \mathscr{P}(\operatorname{Hom}(A_0,B_0)\times\operatorname{Hom}(A_1,B_1))$ for some sets A_0, B_0, A_1, B_1 then

$$\bigsqcap\left\{\frac{a\times^{(L)} b}{(a,b)\in S}\right\} = \bigsqcap\operatorname{dom} S\times^{(L)}\bigsqcap\operatorname{im} S.$$

PROOF.

$$\begin{aligned}
&\bigsqcap\left\{\frac{a\times b}{(a,b)\in S}\right\} = \\
&\bigsqcap\left\{\frac{((\pi_0^{(\operatorname{Dst} a,\operatorname{Dst} b)})^\dagger\circ a\circ\pi_0^{(\operatorname{Src} a,\operatorname{Src} b)})\sqcap((\pi_1^{(\operatorname{Dst} a,\operatorname{Dst} b)})^\dagger\circ b\circ\pi_1^{(\operatorname{Src} a,\operatorname{Src} b)})}{(a,b)\in S}\right\} = \\
&\bigsqcap\left\{\frac{(\pi_0^{(\operatorname{Dst} a,\operatorname{Dst} b)})^\dagger\circ a\circ\pi_0^{(\operatorname{Src} a,\operatorname{Src} b)}}{a\in\operatorname{dom} S}\right\}\sqcap\bigsqcap\left\{\frac{(\pi_1^{(\operatorname{Dst} a,\operatorname{Dst} b)})^\dagger\circ b\circ\pi_1^{(\operatorname{Src} a,\operatorname{Src} b)}}{b\in\operatorname{im} S}\right\} = \\
&\left((\pi_0^{(\operatorname{Dst} a,\operatorname{Dst} b)})^\dagger\circ\bigsqcap\left\{\frac{a}{a\in\operatorname{dom} S}\right\}\circ\pi_0^{(\operatorname{Src} a,\operatorname{Src} b)}\right)\sqcap\left((\pi_1^{(\operatorname{Dst} a,\operatorname{Dst} b)})^\dagger\circ\bigsqcap\left\{\frac{b}{b\in\operatorname{im} S}\right\}\circ\pi_1^{(\operatorname{Src} a,\operatorname{Src} b)}\right) = \\
&\left((\pi_0^{(\operatorname{Dst} a,\operatorname{Dst} b)})^\dagger\circ\left(\bigsqcap\operatorname{dom} S\right)\circ\pi_0^{(\operatorname{Src} a,\operatorname{Src} b)}\right)\sqcap\left((\pi_1^{(\operatorname{Dst} a,\operatorname{Dst} b)})^\dagger\circ\left(\bigsqcap\operatorname{im} S\right)\circ\pi_1^{(\operatorname{Src} a,\operatorname{Src} b)}\right) = \\
&\bigsqcap\operatorname{dom} S\times\bigsqcap\operatorname{im} S.
\end{aligned}$$

□

COROLLARY 1980. $(a_0\times^{(L)} b_0)\sqcap(a_1\times^{(L)} b_1) = (a_0\sqcap a_1)\times^{(L)}(b_0\sqcap b_1)$.

COROLLARY 1981. $a_0\times^{(L)} b_0 \not\asymp a_1\times^{(L)} b_1 \Leftrightarrow a_0\not\asymp a_1\wedge b_0\not\asymp b_1$.