$a, b \in R$. $M(a) = E^{-1}a$; $M(b) = E^{-1}b$; $E^{-1}a = E^{-1}b$; thus $a = b$ because $E^{-1}$ is a bijection.

$a \in R$, $b \notin R$. $M(a) = E^{-1}a$; $M(b) = (t, b)$; $M(a) \in S$; $M(b) \notin S$. Thus $M(a) \neq M(b)$.

$a \notin R$, $b \in R$. Analogous.

$a, b \notin R$. $M(a) = (t, a)$; $M(b) = (t, b)$. Thus $M(a) = M(b)$ implies $a = b$.

THEOREM 1952. $M \circ E = \mathrm{id}_S$.

PROOF. Let $x \in S$. Then $Ex \in R$; $M(Ex) = E^{-1}Ex = x$. □

OBVIOUS 1953. $E = M^{-1}|_S$.

### A.1.2. Existence of primary filtrator.

THEOREM 1954. For every poset $\mathfrak{Z}$ there exists a poset $\mathfrak{A} \supseteq \mathfrak{Z}$ such that $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator.

PROOF. Take $S = \mathfrak{Z}$, $B = \mathfrak{F}$, $E = \uparrow$. By the above there exists an injection $M$ defined on $\mathfrak{F}$ such that $M \circ \uparrow = \mathrm{id}_{\mathfrak{Z}}$.

Take $\mathfrak{A} = \operatorname{im} M$. Order ($\sqsubseteq'$) elements of $\mathfrak{A}$ in such a way that $M : \mathfrak{F}(\mathfrak{Z}) \to \mathfrak{A}$ become order isomorphism. If $x \in \mathfrak{Z}$ then $x = \mathrm{id}_{\mathfrak{Z}}\, x = M \uparrow x \in \operatorname{im} M = \mathfrak{A}$. Thus $\mathfrak{A} \supseteq \mathfrak{Z}$.

If $x \sqsubseteq y$ for elements $x$, $y$ of $\mathfrak{Z}$, then $\uparrow x \sqsubseteq \uparrow y$ and thus $M \uparrow x \sqsubseteq' M \uparrow y$ that is $x \sqsubseteq' y$, so $\mathfrak{Z}$ is a subposet of $\mathfrak{A}$, that is $(\mathfrak{A}, \mathfrak{Z})$ is a filtrator.

It remains to prove that $M$ is an isomorphism between filtrators $(\mathfrak{F}(\mathfrak{Z}), \mathfrak{P})$ and $(\mathfrak{A}, \mathfrak{Z})$. That $M$ is an order isomorphism from $\mathfrak{F}(\mathfrak{Z})$ to $\mathfrak{A}$ is already known. It remains to prove that $M$ maps $\mathfrak{P}$ to $\mathfrak{Z}$. We will instead prove that $M^{-1}$ maps $\mathfrak{Z}$ to $\mathfrak{P}$. Really, $\uparrow x = M^{-1}x$ for every $x \in \mathfrak{Z}$. □