

REMARK 1911. At <http://mathoverflow.net/questions/60925/special-infinitary-relations-and-ultrafilters> there is a proof for arbitrary infinite form, not just for \mathbb{N} .

CONJECTURE 1912. For every family $a = a_{i \in \mathbb{N}}$ of ultrafilters $\prod^{\text{Strd}} a$ is not an atom nor of the poset of staroids neither of the poset of cometary staroids of the form $\lambda i \in \mathbb{N} : \text{Base}(a_i)$.

CONJECTURE 1913. There exists a non-cometary staroid on powersets.

CONJECTURE 1914. There exists a prestaroid which is not a staroid.

CONJECTURE 1915. The set of staroids of the form A^B where A and B are sets is atomic.

CONJECTURE 1916. The set of staroids of the form A^B where A and B are sets is atomistic.

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EXAMPLE 1919. $\text{StarComp}(a, f \sqcup g) \neq \text{StarComp}(a, f) \sqcup \text{StarComp}(a, g)$ in the category of binary relations with star-morphisms for some n -ary relation a and an n -indexed families f and g of functions.

PROOF. Let $n = \{0, 1\}$. Let $\text{GR } a = \{(0, 1), (1, 0)\}$ and $f = [\{(0, 1)\}, \{(1, 0)\}]$, $g = [\{(1, 0)\}, \{(0, 1)\}]$.

For every $\{0, 1\}$ -indexed family of μ of functions:

$$L \in \text{StarComp}(a, \mu) \Leftrightarrow \exists y \in a : (y_0 \mu_0 L_0 \wedge y_1 \mu_1 L_1) \Leftrightarrow \exists y_0 \in \text{dom } \mu_0, y_1 \in \text{dom } \mu_1 : (y_0 \mu_0 L_0 \wedge y_1 \mu_1 L_1)$$

for every n -ary relation μ .

Consequently

$$L \in \text{StarComp}(a, f) \Leftrightarrow L_0 = 1 \wedge L_1 = 0 \Leftrightarrow L = (1, 0)$$

that is $\text{StarComp}(a, f) = \{(1, 0)\}$. Similarly

$$\text{StarComp}(a, g) = \{(0, 1)\}.$$

Also

$$L \in \text{StarComp}(a, f \sqcup g) \Leftrightarrow \exists y_0, y_1 \in \{0, 1\} : ((y_0 f_0 L_0 \vee y_0 g_0 L_0) \wedge (y_1 f_1 L_1 \vee y_1 g_1 L_1)).$$

Thus

$$\text{StarComp}(a, f \sqcup g) = \{(0, 1), (1, 0), (0, 0), (1, 1)\}.$$

□

COROLLARY 1920. The above inequality is possible also for star-morphisms of functors and star-morphisms of reloids.

PROOF. Because finitary functors and reloids between finite sets are essentially the same as finitary relations and our proof above works for binary relations. □

The following example shows that the theorem 1861 can't be strengthened:

EXAMPLE 1921. For some multifunctor f on powersets complete in argument k the following formula is false:

$$\langle f \rangle_i(L \cup \{(k, \sqcup X)\}) = \sqcup_{x \in X} \langle f \rangle_i(L \cup \{(k, x)\}) \text{ for every } X \in \mathcal{P} \mathfrak{Z}_k, L \in \prod_{i \in (\text{arity } f) \setminus \{k, l\}} \mathcal{F}_i.$$