Let $f \sqsupseteq \mathrm{id}^{\mathsf{Strd}}_{a[n]}$ for every $a \in \operatorname{atoms}\mathcal{A}$. Then $\forall L \in \operatorname{GR}\mathrm{id}^{\mathsf{Strd}}_{a[n]} : L \in \operatorname{GR} f$ that is

$$\forall L \in \operatorname{form} f : \left(\prod_{i\in n}^{\mathfrak{Z}} L_i \not\asymp a \Rightarrow L \in \operatorname{GR} f \right).$$

But $\exists a \in \operatorname{atoms}\mathcal{A} : \prod^{\mathfrak{Z}}_{i\in n} L_i \not\asymp a \Leftarrow \prod^{\mathfrak{Z}}_{i\in n} L_i \not\asymp \mathcal{A} \Leftrightarrow L \in \mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$.

So $L \in \mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]} \Rightarrow L \in \operatorname{GR} f$. Thus $f \sqsupseteq \mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$. □

21.19.8. Finite case.

THEOREM 1904. Let n be a finite set.

1°. $\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]} = \downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ if $\mathfrak{A}$ and $\mathfrak{Z}$ are meet-semilattices and $(\mathfrak{A}, \mathfrak{Z})$ is a binarily meet-closed filtrator.

2°. $\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]} = \upuparrows \mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ if $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator over a distributive lattice.

PROOF.

1°.

$$L \in \operatorname{GR} \downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]} \Leftrightarrow L \in \operatorname{GR}\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]} \Leftrightarrow \operatorname{MEET}\left(\left\{ \frac{L_i}{i \in n} \right\} \cup \{\mathcal{A}\} \right) \Leftrightarrow$$
$$\prod_{i\in n}^{\mathfrak{A}} L_i \sqcap \mathcal{A} \neq 0 \Leftrightarrow (\text{by finiteness}) \Leftrightarrow \prod_{i\in n}^{\mathfrak{Z}} L_i \sqcap \mathcal{A} \neq 0 \Leftrightarrow L \in \operatorname{GR}\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$$

for every $L \in \prod \mathfrak{Z}$.

2°.

$$L \in \operatorname{GR} \upuparrows \mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]} \Leftrightarrow \operatorname{up} L \subseteq \operatorname{GR}\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]} \Leftrightarrow \forall K \in \operatorname{up} L : K \in \operatorname{GR}\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]} \Leftrightarrow$$
$$\forall K \in \operatorname{up} L : \prod_{i\in n}^{\mathfrak{Z}} K_i \in \partial\mathcal{A} \Leftrightarrow \forall K \in \operatorname{up} L : \prod_{i\in n}^{\mathfrak{Z}} K_i \not\asymp \mathcal{A} \Leftrightarrow$$
$$(\text{by finiteness and theorem } 532) \Leftrightarrow$$
$$\forall K \in \operatorname{up} L : \prod_{i\in n}^{\mathfrak{A}} K_i \not\asymp \mathcal{A} \Leftrightarrow \mathcal{A} \in \bigcap \langle\star\rangle^* \left\{ \frac{\prod^{\mathfrak{A}}_{i\in n} K_i}{K \in \operatorname{up} L} \right\} \Leftrightarrow$$
$$(\text{by the formula for finite meet of filters, theorem } 520) \Leftrightarrow$$
$$\mathcal{A} \in \bigcap \langle\star\rangle^* \operatorname{up} \prod_{i\in n}^{\mathfrak{A}} L_i \Leftrightarrow \forall K \in \operatorname{up} \prod_{i\in n}^{\mathfrak{A}} L_i : \mathcal{A} \in \star K \Leftrightarrow \forall K \in \operatorname{up} \prod_{i\in n}^{\mathfrak{A}} L_i : \mathcal{A} \not\asymp K \Leftrightarrow$$
$$(\text{by separability of core, theorem } 534) \Leftrightarrow$$
$$\prod_{i\in n}^{\mathfrak{A}} L_i \not\asymp \mathcal{A} \Leftrightarrow L \in \mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}.$$

□

PROPOSITION 1905. Let $(\mathfrak{A}, \mathfrak{Z})$ be a binarily meet closed filtrator whose core is a meet-semilattice. $\downdownarrows \mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ and $\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ are the same for finite n.

PROOF. Because $\prod^{\mathfrak{Z}}_{i\in\operatorname{dom} L} L_i = \prod^{\mathfrak{A}}_{i\in\operatorname{dom} L} L_i$ for finitary L. □

21.20. Counter-examples

EXAMPLE 1906. $\upuparrows\downdownarrows f \neq f$ for some staroid f whose form is an indexed family of filters on a set.