REMARK 1900. $\sqsubseteq$ on the diagram means inequality which can become strict for some A and n.

21.19.7. Identity staroids represented as meets and joins.

PROPOSITION 1901. $\mathrm{id}^{\mathsf{Strd}}_{a[n]} = \bigsqcap^{\mathrm{Anch}}_{A\in\mathrm{up}\,a} \mathrm{id}_{A[n]} = \bigsqcap^{\mathsf{Strd}}_{A\in\mathrm{up}\,a} \mathrm{id}_{A[n]}$ for every filter a on a powerset.

PROOF. Since $\mathrm{id}^{\mathsf{Strd}}_{a[n]}$ is a staroid (proposition 1872), it's enough to prove that $\mathrm{id}^{\mathsf{Strd}}_{a[n]}$ is the greatest lower bound of $\left\{ \frac{\uparrow^{\mathsf{Strd}}\mathrm{id}_{A[n]}}{A\in\mathrm{up}\,a} \right\}$.

That $\mathrm{id}^{\mathsf{Strd}}_{a[n]} \sqsubseteq \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]}$ for every $A \in \mathrm{up}\,a$ is obvious.

Let $f \sqsubseteq \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]}$ for every $A \in \mathrm{up}\,a$.

$$L \in \mathrm{GR}\, f \Rightarrow \forall A \in \mathrm{up}\,a : L \in \mathrm{GR} \uparrow^{\mathsf{Strd}} \mathrm{id}_{A[n]} \Leftrightarrow$$
$$\forall A \in \mathrm{up}\,a : \prod L \not\asymp \mathrm{id}_{A[n]} \Leftrightarrow \forall A \in \mathrm{up}\,a : \bigsqcap_{i\in n}^{\mathfrak{Z}} L_i \not\asymp A \Rightarrow$$
$$\forall A \in \mathrm{up}\,a : \bigsqcap_{i\in n}^{\mathfrak{A}} L_i \not\asymp A \Rightarrow \bigsqcap_{i\in n}^{\mathfrak{A}} L_i \not\asymp a \Rightarrow L \in \mathrm{GR}\,\mathrm{id}^{\mathsf{Strd}}_{a[n]}.$$

Thus $f \sqsubseteq \mathrm{id}^{\mathsf{Strd}}_{a[n]}$. ∎

PROPOSITION 1902. $\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]} = \bigsqcup_{a\in\mathrm{atoms}\,\mathcal{A}} \mathrm{ID}^{\mathsf{Strd}}_{a[n]} = \bigsqcup_{a\in\mathrm{atoms}\,\mathcal{A}} a^n_{\mathsf{Strd}}$ where the join may be taken on every of the following posets: anchored relations, staroids, completary staroids, provided that $\mathcal{A}$ is a filter on a set.

PROOF. $\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ is a completary staroid (proposition 1873). Thus, it's enough to prove that $\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ is the lowest upper bound of $\left\{ \frac{\mathrm{ID}^{\mathsf{Strd}}_{a[n]}}{a\in\mathrm{atoms}\,\mathcal{A}} \right\}$ (also use the fact that $\mathrm{ID}^{\mathsf{Strd}}_{a[n]} = a^n_{\mathsf{Strd}}$).

$\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]} \sqsupseteq \mathrm{ID}^{\mathsf{Strd}}_{a[n]}$ for every $a \in \mathrm{atoms}\,\mathcal{A}$ is obvious.

Let $f \sqsupseteq \mathrm{ID}^{\mathsf{Strd}}_{a[n]}$ for every $a \in \mathrm{atoms}\,\mathcal{A}$. Then $\forall L \in \mathrm{GR}\,\mathrm{ID}^{\mathsf{Strd}}_{a[n]} : L \in \mathrm{GR}\, f$ that is

$$\forall L \in \mathrm{form}\, f : \left(\mathrm{MEET}\left(\left\{ \frac{L_i}{i\in n} \right\} \cup \{a\} \right) \Rightarrow L \in \mathrm{GR}\, f \right).$$

But

$$\exists a \in \mathrm{atoms}\,\mathcal{A} : \mathrm{MEET}\left(\left\{ \frac{L_i}{i\in n} \right\} \cup \{a\} \right) \Leftrightarrow \exists a \in \mathrm{atoms}\,\mathcal{A} : \bigsqcap_{i\in n}^{\mathfrak{A}} L_i \not\asymp a \Leftarrow$$
$$\bigsqcap_{i\in n}^{\mathfrak{A}} L_i \not\asymp \mathcal{A} \Leftrightarrow L \in \mathrm{GR}\,\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}.$$

So $L \in \mathrm{GR}\,\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]} \Rightarrow L \in \mathrm{GR}\, f$. Thus $f \sqsupseteq \mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}$. ∎

PROPOSITION 1903. $\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]} = \bigsqcup_{a\in\mathrm{atoms}\,\mathcal{A}} \mathrm{id}^{\mathsf{Strd}}_{a[n]}$ where the meet may be taken on every of the following posets: anchored relations, staroids, provided that $\mathcal{A}$ is a filter on a set.

PROOF. Since $\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ is a staroid (proposition 1872), it's enough to prove the result for join on anchored relations.

$\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]} \sqsupseteq \mathrm{id}^{\mathsf{Strd}}_{a[n]}$ for every $a \in \mathrm{atoms}\,\mathcal{A}$ is obvious.