PROOF. That $L \notin \operatorname{GR} \operatorname{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ if $L_k = \bot$ for some $k \in n$ is obvious. It remains to prove

$L \cup \{(k, X \sqcup Y)\} \in \operatorname{GR} \operatorname{id}^{\mathsf{Strd}}_{\mathcal{A}[n]} \Leftrightarrow L \cup \{(k, X)\} \in \operatorname{GR} \operatorname{id}^{\mathsf{Strd}}_{\mathcal{A}[n]} \vee L \cup \{(k, Y)\} \in \operatorname{GR} \operatorname{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$.

It is equivalent to

$$\prod^{\mathfrak{Z}}_{i \in n \setminus \{k\}} L_i \sqcap (X \sqcup Y) \not\asymp \mathcal{A} \Leftrightarrow \prod^{\mathfrak{Z}}_{i \in n \setminus \{k\}} L_i \sqcap X \not\asymp \mathcal{A} \vee \prod^{\mathfrak{Z}}_{i \in n \setminus \{k\}} L_i \sqcap Y \not\asymp \mathcal{A}.$$

Really,

$$\prod^{\mathfrak{Z}}_{i \in n \setminus \{k\}} L_i \sqcap (X \sqcup Y) \not\asymp \mathcal{A} \Leftrightarrow \prod^{\mathfrak{Z}}_{i \in n \setminus \{k\}} ((L_i \sqcap X) \sqcup (L_i \sqcap Y)) \Leftrightarrow \left(\prod^{\mathfrak{Z}}_{i \in n \setminus \{k\}} L_i \sqcap X \right) \sqcup \left(\prod^{\mathfrak{Z}}_{i \in n \setminus \{k\}} L_i \sqcap Y \right) \not\asymp \mathcal{A} \Leftrightarrow \prod^{\mathfrak{Z}}_{i \in n \setminus \{k\}} L_i \sqcap X \not\asymp \mathcal{A} \vee \prod^{\mathfrak{Z}}_{i \in n \setminus \{k\}} L_i \sqcap Y \not\asymp \mathcal{A}.$$

□

PROPOSITION 1873. Let $(\mathfrak{A}, \mathfrak{Z})$ be a primary filtrator over a boolean lattice. $\operatorname{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ is a completary staroid for every $\mathcal{A} \in \mathfrak{A}$.

PROOF. $\star\mathcal{A}$ is a free star by theorem 611.

$$L_0 \sqcup L_1 \in \operatorname{GR} \operatorname{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]} \Leftrightarrow \forall i \in n : (L_0 \sqcup L_1) i \in \star\mathcal{A} \Leftrightarrow \forall i \in n : L_0 i \sqcup L_1 i \in \star\mathcal{A} \Leftrightarrow \forall i \in n : (L_0 i \in \star\mathcal{A} \vee L_1 i \in \star\mathcal{A}) \Leftrightarrow \exists c \in \{0,1\}^n \forall i \in n : L_{c(i)} i \in \star\mathcal{A} \Leftrightarrow \exists c \in \{0,1\}^n : (\lambda i \in n : L_{c(i)} i) \in \operatorname{GR} \operatorname{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}.$$

□

LEMMA 1874. $X \in \operatorname{GR} \operatorname{id}^{\mathsf{Strd}}_{\mathcal{A}[n]} \Leftrightarrow \operatorname{Cor}' \prod^{\mathfrak{A}}_{i \in n} X_i \not\asymp \mathcal{A}$ for a join-closed filtrator $(\mathfrak{A}, \mathfrak{Z})$ such that both $\mathfrak{A}$ and $\mathfrak{Z}$ are complete lattices, provided that $\mathcal{A} \in \mathfrak{A}$.

PROOF. $X \in \operatorname{GR} \operatorname{id}^{\mathsf{Strd}}_{\mathcal{A}[n]} \Leftrightarrow \prod^{\mathfrak{Z}}_{i \in n} X_i \not\asymp \mathcal{A} \Leftrightarrow \operatorname{Cor}' \prod^{\mathfrak{A}}_{i \in n} X_i \not\asymp \mathcal{A}$ (theorem 599). □

CONJECTURE 1875. $\operatorname{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ is a completary staroid for every set-theoretic filter $\mathcal{A}$.

CONJECTURE 1876. $\upuparrows \operatorname{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ is a completary staroid if $\mathcal{A}$ is a filter on a set and n is an index set.

21.19.4. Special case of sets and filters.

PROPOSITION 1877. $\uparrow^{\mathfrak{Z}^n} X \in \operatorname{GR} \operatorname{id}^{\mathsf{Strd}}_{a[n]} \Leftrightarrow \forall A \in a : \prod X \not\asymp \operatorname{id}_{A[n]}$ for every filter a on a powerset and index set n.

PROOF.

$$\forall A \in a : \prod X \not\asymp \operatorname{id}_{A[n]} \Leftrightarrow \forall A \in a : \bigcap_{i \in n} X_i \cap A \neq \emptyset \Leftrightarrow \forall A \in a : \prod^{\mathfrak{Z}}_{i \in n} X_i \not\asymp A \Leftrightarrow$$

$$\forall A \in a : \prod^{\mathfrak{Z}}_{i \in n} X_i \not\asymp^{\mathfrak{A}} A \Leftrightarrow \prod^{\mathfrak{Z}}_{i \in n} \uparrow^{\mathfrak{Z}} X_i \not\asymp^{\mathfrak{A}} a \Leftrightarrow \prod^{\mathfrak{Z}}_{i \in n} (\uparrow^{\mathfrak{Z}^n} X)_i \not\asymp^{\mathfrak{A}} a \Leftrightarrow \uparrow^{\mathfrak{Z}^n} X \in \operatorname{GR} \operatorname{id}_{a[n]}.$$

□