Definition 1866. Staroid $\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ (for any $\mathcal{A}\in\mathfrak{A}$ where $\mathfrak{A}$ is a poset) is defined by the formulas:

$$\operatorname{form}\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}=\mathfrak{A}^n;\quad \mathcal{L}\in\operatorname{GR}\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}\Leftrightarrow\operatorname{MEET}\left(\left\{\frac{\mathcal{L}_i}{i\in n}\right\}\cup\{\mathcal{A}\}\right).$$

Obvious 1867. If $\mathfrak{A}$ is complete lattice, then $\mathcal{L}\in\operatorname{GR}\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}\Leftrightarrow\bigsqcap\mathcal{L}\not\asymp\mathcal{A}$.

Obvious 1868. If $\mathfrak{A}$ is complete lattice and a is an atom, then $\mathcal{L}\in\operatorname{GR}\mathrm{ID}^{\mathsf{Strd}}_{a[n]}\Leftrightarrow\bigsqcap\mathcal{L}\sqsupseteq a$.

Obvious 1869. If $\mathfrak{A}$ is a complete lattice then there exists a multifuncoid $\Lambda\,\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ such that $\langle\Lambda\,\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}\rangle_k L=\bigsqcap_{i\in n}L_i\sqcap\mathcal{A}$ for every $k\in n$, $L\in\mathfrak{A}^{n\setminus\{k\}}$.

Proposition 1870. Let $(\mathfrak{A},\mathfrak{Z})$ be a meet-closed filtrator and $\mathfrak{Z}$ be a complete lattice and $\mathfrak{A}$ be a meet-semilattice. There exists a multifuncoid $\Lambda\,\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ such that $\langle\Lambda\,\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}\rangle_k L=\bigsqcap^{\mathfrak{Z}}_{i\in n}L_i\sqcap^{\mathfrak{A}}\mathcal{A}$ for every $k\in n$, $L\in\mathfrak{Z}^{n\setminus\{k\}}$.

Proof. We need to prove that $L\cup\{(k,X)\}\in\operatorname{GR}\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}\Leftrightarrow\bigsqcap^{\mathfrak{Z}}_{i\in n}L_i\sqcap^{\mathfrak{A}}\mathcal{A}\not\asymp^{\mathfrak{A}}X$. But

$$\bigsqcap^{\mathfrak{Z}}_{i\in n}L_i\sqcap^{\mathfrak{A}}\mathcal{A}\not\asymp^{\mathfrak{A}}X\Leftrightarrow\bigsqcap^{\mathfrak{Z}}_{i\in n}L_i\sqcap^{\mathfrak{A}}X\not\asymp^{\mathfrak{A}}\mathcal{A}\Leftrightarrow$$
$$\bigsqcap^{\mathfrak{Z}}_{i\in n}(L\cup\{(k,X)\})_i\not\asymp^{\mathfrak{A}}\mathcal{A}\Leftrightarrow L\cup\{(k,X)\}\in\operatorname{GR}\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}.$$

□

21.19.3. Identities are staroids.

Proposition 1871. Let $\mathfrak{A}$ be a complete meet infinite distributive lattice and $\mathcal{A}\in\mathfrak{A}$. Then $\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ is a staroid.

Proof. That $L\notin\operatorname{GR}\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ if $L_k=\bot$ for some $k\in n$ is obvious. It remains to prove

$$L\cup\{(k,X\sqcup Y)\}\in\operatorname{GR}\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}\Leftrightarrow L\cup\{(k,X)\}\in\operatorname{GR}\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}\vee L\cup\{(k,Y)\}\in\operatorname{GR}\mathrm{ID}^{\mathsf{Strd}}_{\mathcal{A}[n]}.$$

It is equivalent to

$$\bigsqcap_{i\in n\setminus\{k\}}L_i\sqcap(X\sqcup Y)\not\asymp\mathcal{A}\Leftrightarrow\bigsqcap_{i\in n\setminus\{k\}}L_i\sqcap X\not\asymp\mathcal{A}\vee\bigsqcap_{i\in n\setminus\{k\}}L_i\sqcap Y\not\asymp\mathcal{A}.$$

Really,

$$\bigsqcap_{i\in n\setminus\{k\}}L_i\sqcap(X\sqcup Y)\not\asymp\mathcal{A}\Leftrightarrow\bigsqcap_{i\in n\setminus\{k\}}((L_i\sqcap X)\sqcup(L_i\sqcap Y))\not\asymp\mathcal{A}\Leftrightarrow$$
$$\left(\bigsqcap_{i\in n\setminus\{k\}}L_i\sqcap X\right)\sqcup\left(\bigsqcap_{i\in n\setminus\{k\}}L_i\sqcap Y\right)\not\asymp\mathcal{A}\Leftrightarrow$$
$$\bigsqcap_{i\in n\setminus\{k\}}L_i\sqcap X\not\asymp\mathcal{A}\vee\bigsqcap_{i\in n\setminus\{k\}}L_i\sqcap Y\not\asymp\mathcal{A}.$$

□

Proposition 1872. Let $(\mathfrak{A},\mathfrak{Z})$ be a starrish filtrator over a complete meet infinite distributive lattice and $\mathcal{A}\in\mathfrak{A}$. Then $\mathrm{id}^{\mathsf{Strd}}_{\mathcal{A}[n]}$ is a staroid.