21.18.1. More on free stars and principal free stars.

PROPOSITION 1850. The following is an implications tuple:

1°. $(\mathfrak{A}, \mathfrak{Z})$ is a powerset filtrator.
2°. $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator.
3°. $(\mathfrak{A}, \mathfrak{Z})$ is a filtrator.
4°. $\partial\mathcal{F} = \downdownarrows \star\mathcal{F}$ for every $\mathcal{F} \in \mathfrak{A}$.

PROOF.

1°⇒2°. Obvious.
2°⇒3°. Obvious.
3°⇒4°. $X \in \partial\mathcal{F} \Leftrightarrow X \not\asymp^{\mathfrak{A}} \mathcal{F} \Leftrightarrow X \in \star\mathcal{F} \Leftrightarrow X \in \downdownarrows \star\mathcal{F}$ for every $X \in \mathfrak{Z}$.

□

PROPOSITION 1851. The following is an implications tuple:

1°. $(\mathfrak{A}, \mathfrak{Z})$ is a powerset filtrator.
2°. $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator over a meet-semilattice with least element.
3°. $(\mathfrak{A}, \mathfrak{Z})$ is a filtrator with separable core.
4°. $\star\mathcal{F} = \upuparrows \partial\mathcal{F}$ for every $\mathcal{F} \in \mathfrak{A}$.

PROOF.

1°⇒2°. Obvious.
2°⇒3°. Theorem 534.
3°⇒4°. $\mathcal{X} \in \upuparrows \partial\mathcal{F} \Leftrightarrow \operatorname{up}\mathcal{X} \subseteq \partial\mathcal{F} \Leftrightarrow \forall X \in \operatorname{up}\mathcal{X} : X \not\asymp \mathcal{F} \Leftrightarrow \mathcal{X} \not\asymp \mathcal{F} \Leftrightarrow \mathcal{X} \in \star\mathcal{F}$.

□

PROPOSITION 1852. The following is an implications tuple:

1°. $(\mathfrak{A}, \mathfrak{Z})$ is a powerset filtrator.
2°. $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator over a complete boolean lattice.
3°. $(\mathfrak{A}, \mathfrak{Z})$ is a down-aligned, with join-closed, binarily meet-closed and separable core which is a complete boolean lattice.
4°. The following conditions are equivalent for any $\mathcal{F} \in \mathfrak{A}$:
   (a) $\mathcal{F} \in \mathfrak{Z}$.
   (b) $\partial\mathcal{F}$ is a principal free star on $\mathfrak{Z}$.
   (c) $\star\mathcal{F}$ is a principal free star on $\mathfrak{A}$.

PROOF.

1°⇒2°. Obvious.
2°⇒3°. The filtrator $(\mathfrak{A}, \mathfrak{Z})$ is with with join-closed core by theorem 531, binarily meet-closed core by corollary 533, with separable core by theorem 534.
3°⇒4°.
   4°a⇒4°b. That $\partial\mathcal{F}$ does not contain the least element is obvious. That $\partial\mathcal{F}$ is an upper set is obvious. So it remains to apply theorem 580.
   4°b⇒4°c. That $\star\mathcal{F}$ does not contain the least element is obvious. That $\star\mathcal{F}$ is an upper set is obvious. So it remains to apply theorem 580.
   4°c⇒4°a. Apply theorem 580.

□

PROPOSITION 1853. The following is an implications tuple:

1°. $(\mathfrak{A}, \mathfrak{Z})$ is a powerset filtrator.
2°. $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator over a join-semilattice.
3°. The filtrator $(\mathfrak{A}, \mathfrak{Z})$ is weakly down-aligned and with binarily join-closed core and $\mathfrak{Z}$ is a join-semilattice.
4°. If S is a free star on $\mathfrak{A}$ then $\downdownarrows S$ is a free star on $\mathfrak{Z}$.