OBVIOUS 1845. Pseudofuncoid is just a staroid of the form $(\mathscr{F}(A), \mathscr{F}(B))$.

OBVIOUS 1846. $[f]$ is a pseudofuncoid for every funcoid f.

EXAMPLE 1847. If A and B are infinite sets, then there exist two different pseudofuncoids f and g from A to B such that $f \cap (\mathscr{T}A \times \mathscr{T}B) = g \cap (\mathscr{T}A \times \mathscr{T}B) = [c] \cap (\mathscr{T}A \times \mathscr{T}B)$ for some funcoid c.

REMARK 1848. Considering a pseudofuncoid f as a staroid, we get $f \cap (\mathscr{T}A \times \mathscr{T}B) = \Downarrow f$.

PROOF. Take

$$f = \left\{ \frac{(\mathcal{X}, \mathcal{Y})}{\mathcal{X} \in \mathscr{F}(A), \mathcal{Y} \in \mathscr{F}(B), \bigcap \mathcal{X} \text{ and } \bigcap \mathcal{Y} \text{ are infinite}} \right\}$$

and

$$g = f \cup \left\{ \frac{(\mathcal{X}, \mathcal{Y})}{\mathcal{X} \in \mathscr{F}(A), \mathcal{Y} \in \mathscr{F}(B), \mathcal{X} \sqsupseteq a, \mathcal{Y} \sqsupseteq b} \right\}$$

where a and b are nontrivial ultrafilters on A and B correspondingly, c is the funcoid defined by the relation

$$[c]^* = \delta = \left\{ \frac{(X, Y)}{X \in \mathscr{P}A, Y \in \mathscr{P}B, X \text{ and } Y \text{ are infinite}} \right\}.$$

First prove that f is a pseudofuncoid. The formulas $\neg(\mathcal{I}\ f\ \bot)$ and $\neg(\bot\ f\ \mathcal{I})$ are obvious. We have

$$\mathcal{I} \sqcup \mathcal{J}\ f\ \mathcal{K} \Leftrightarrow \bigcap(\mathcal{I} \sqcup \mathcal{J}) \text{ and } \bigcap \mathcal{Y} \text{ are infinite} \Leftrightarrow$$
$$\bigcap \mathcal{I} \cup \bigcap \mathcal{J} \text{ and } \bigcap \mathcal{Y} \text{ are infinite} \Leftrightarrow \left(\bigcap \mathcal{I} \text{ or } \bigcap \mathcal{J} \text{ is infinite}\right) \wedge \bigcap \mathcal{Y} \text{ is infinite} \Leftrightarrow$$
$$\left(\bigcap \mathcal{I} \text{ and } \bigcap \mathcal{Y} \text{ are infinite}\right) \vee \left(\bigcap \mathcal{J} \text{ and } \bigcap \mathcal{Y} \text{ are infinite}\right) \Leftrightarrow$$
$$\mathcal{I}\ f\ \mathcal{K} \vee \mathcal{J}\ f\ \mathcal{K}.$$

Similarly $\mathcal{K}\ f\ \mathcal{I} \sqcup \mathcal{J} \Leftrightarrow \mathcal{K}\ f\ \mathcal{I} \vee \mathcal{K}\ f\ \mathcal{J}$. So f is a pseudofuncoid.

Let now prove that g is a pseudofuncoid. The formulas $\neg(\mathcal{I}\ g\ \bot)$ and $\neg(\bot\ g\ \mathcal{I})$ are obvious. Let $\mathcal{I} \sqcup \mathcal{J}\ g\ \mathcal{K}$. Then either $\mathcal{I} \sqcup \mathcal{J}\ f\ \mathcal{K}$ and then $\mathcal{I} \sqcup \mathcal{J}\ g\ \mathcal{K}$ or $\mathcal{I} \sqcup \mathcal{J} \sqsupseteq a$ and then $\mathcal{I} \sqsupseteq a \vee \mathcal{J} \sqsupseteq a$ thus having $\mathcal{I}\ g\ \mathcal{K} \vee \mathcal{J}\ g\ \mathcal{K}$. So $\mathcal{I} \sqcup \mathcal{J}\ g\ \mathcal{K} \Rightarrow \mathcal{I}\ g\ \mathcal{K} \vee \mathcal{J}\ g\ \mathcal{K}$. The reverse implication is obvious. We have $\mathcal{I} \sqcup \mathcal{J}\ g\ \mathcal{K} \Leftrightarrow \mathcal{I}\ g\ \mathcal{K} \vee \mathcal{J}\ g\ \mathcal{K}$ and similarly $\mathcal{K}\ g\ \mathcal{I} \sqcup \mathcal{J} \Leftrightarrow \mathcal{K}\ g\ \mathcal{I} \vee \mathcal{K}\ g\ \mathcal{J}$. So g is a pseudofuncoid.

Obviously $f \neq g$ ($a\ g\ b$ but not $a\ f\ b$).

It remains to prove $f \cap (\mathscr{T}A \times \mathscr{T}B) = g \cap (\mathscr{T}A \times \mathscr{T}B) = [c] \cap (\mathscr{T}A \times \mathscr{T}B)$. Really, $f \cap (\mathscr{T}A \times \mathscr{T}B) = [c] \cap (\mathscr{T}A \times \mathscr{T}B)$ is obvious. If $(\uparrow^A X, \uparrow^B Y) \in g \cap (\mathscr{T}A \times \mathscr{T}B)$ then either $(\uparrow^A X, \uparrow^B Y) \in f \cap (\mathscr{T}A \times \mathscr{T}B)$ or $X \in \operatorname{up} a$, $Y \in \operatorname{up} b$, so X and Y are infinite and thus $(\uparrow^A X, \uparrow^B Y) \in f \cap (\mathscr{T}A \times \mathscr{T}B)$. So $g \cap (\mathscr{T}A \times \mathscr{T}B) = f \cap (\mathscr{T}A \times \mathscr{T}B)$. □

REMARK 1849. The above counter-example shows that pseudofuncoids (and more generally, any staroids on filters) are "second class" objects, they are not full-fledged because they don't bijectively correspond to funcoids and the elegant funcoids theory does not apply to them.

From the above it follows that staroids on filters do not correspond (by restriction) to staroids on principal filters (or staroids on sets).