PROOF.

$$\begin{aligned}
\mathcal{L}_i \not\asymp \langle \upuparrows f\rangle^* \mathcal{L}|_{(\operatorname{dom}\mathcal{L})\setminus\{i\}} &\Leftrightarrow \\
\mathcal{L}_i \not\asymp \langle f\rangle \mathcal{L}|_{(\operatorname{dom}\mathcal{L})\setminus\{i\}} &\Leftrightarrow \\
\mathcal{L}_i \not\asymp \bigsqcap_{X\in\operatorname{up}\mathcal{L}|_{(\operatorname{dom}\mathcal{L})\setminus\{i\}}} \langle f\rangle^* X &\Leftrightarrow \\
\mathcal{L}_i \sqcap \bigsqcap_{X\in\operatorname{up}\mathcal{L}|_{(\operatorname{dom}\mathcal{L})\setminus\{i\}}} \langle f\rangle^* X \neq \bot &\Leftrightarrow \\
\bigsqcap_{X\in\operatorname{up}\mathcal{L}|_{(\operatorname{dom}\mathcal{L})\setminus\{i\}}} \langle \mathcal{L}_i\sqcap\rangle^* \langle f\rangle^* X \neq \bot &\Leftrightarrow \\
\bigsqcap \left\{ \frac{\mathcal{L}_i \sqcap \langle f\rangle^* X}{X\in\operatorname{up}\mathcal{L}|_{(\operatorname{dom}\mathcal{L})\setminus\{i\}}} \right\} \neq \bot &\Leftrightarrow \quad (*) \\
\bot \notin \left\{ \frac{\mathcal{L}_i \sqcap \langle f\rangle^* X}{X\in\operatorname{up}\mathcal{L}|_{(\operatorname{dom}\mathcal{L})\setminus\{i\}}} \right\} &\Leftrightarrow \\
\forall X\in\operatorname{up}\mathcal{L}|_{(\operatorname{dom}\mathcal{L})\setminus\{i\}} : \mathcal{L}_i \sqcap \langle f\rangle^* X \neq \bot &\Leftrightarrow \quad (**) \\
\forall L\in\operatorname{up}\mathcal{L} : \langle f\rangle^* L|_{\operatorname{dom}\mathcal{L}} \sqcap L_i \neq \bot &\Leftrightarrow \\
\forall L\in\operatorname{up}\mathcal{L} : L_i \not\asymp \langle f\rangle^* L|_{\operatorname{dom}\mathcal{L}}&.
\end{aligned}$$

(*) because $\left\{ \frac{\mathcal{L}_i \sqcap \langle f\rangle^* X}{X\in\operatorname{up}\mathcal{L}|_{(\operatorname{dom}\mathcal{L})\setminus\{i\}}} \right\}$ is a filter base (by lemma 1830) of the filter $\bigsqcap \left\{ \frac{\mathcal{L}_i \sqcap \langle f\rangle^* X}{X\in\operatorname{up}\mathcal{L}|_{(\operatorname{dom}\mathcal{L})\setminus\{i\}}} \right\}$.

(**) by theorem 534. □

PROPOSITION 1836. $\upuparrows f$ is a square multifuncoid, if every $((\operatorname{base} f)_i, (\operatorname{core} f)_i)$ is a primary filtrator over a bounded meet-semilattice.

PROOF. Our filtrators are with complete base by corollary 515.

$\mathcal{L}_i \not\asymp \langle \upuparrows f\rangle^* \mathcal{L}|_{(\operatorname{dom}\mathcal{L})\setminus\{i\}} \Leftrightarrow \forall L\in\operatorname{up}\mathcal{L} : L_i \not\asymp \langle f\rangle^* L|_{(\operatorname{dom}\mathcal{L})\setminus\{i\}}$ by the lemma.

Similarly $\mathcal{L}_j \not\asymp \langle \upuparrows f\rangle^* \mathcal{L}|_{(\operatorname{dom}\mathcal{L})\setminus\{j\}} \Leftrightarrow \forall L\in\operatorname{up}\mathcal{L} : L_j \not\asymp \langle f\rangle^* L|_{(\operatorname{dom}\mathcal{L})\setminus\{j\}}$. So $\mathcal{L}_i \not\asymp \langle \upuparrows f\rangle^* \mathcal{L}|_{(\operatorname{dom}\mathcal{L})\setminus\{i\}} \Leftrightarrow \mathcal{L}_j \not\asymp \langle \upuparrows f\rangle^* \mathcal{L}|_{(\operatorname{dom}\mathcal{L})\setminus\{j\}}$ because $L_i \not\asymp \langle f\rangle^* L|_{(\operatorname{dom}\mathcal{L})\setminus\{i\}} \Leftrightarrow L_j \not\asymp \langle f\rangle^* L|_{(\operatorname{dom}\mathcal{L})\setminus\{j\}}$. □

PROPOSITION 1837. $[\upuparrows f]^* = [f]$ if every $((\operatorname{base} f)_i, (\operatorname{core} f)_i)$ is a primary filtrator over a bounded meet-semilattice.

PROOF. Our filtrators are with complete base by corollary 515.

$$\begin{aligned}
\mathcal{L} \in [\upuparrows f]^* &\Leftrightarrow \\
\mathcal{L}_i \not\asymp \langle \upuparrows f\rangle^* \mathcal{L}|_{(\operatorname{dom}\mathcal{L})\setminus\{i\}} &\Leftrightarrow \quad \text{(by the lemma)} \\
\forall L\in\operatorname{up}\mathcal{L} : L_i \not\asymp \langle f\rangle^* L|_{(\operatorname{dom} L)\setminus\{i\}} &\Leftrightarrow \\
\forall L\in\operatorname{up}\mathcal{L} : L \in [f]^* &\Leftrightarrow \\
\mathcal{L} \in [f]&.
\end{aligned}$$

□

PROPOSITION 1838. $\mathcal{L} \in [f] \Leftrightarrow \mathcal{L}_i \not\asymp \langle f\rangle \mathcal{L}|_{(\operatorname{dom}\mathcal{L})\setminus\{i\}}$ if every $((\operatorname{base} f)_i, (\operatorname{core} f)_i)$ is a primary filtrator over a bounded meet-semilattice.

PROOF. Our filtrators are with complete base by corollary 515.

The theorem holds because $\upuparrows f$ is a multifuncoid and $[f] = [\upuparrows f]^*$ and $\langle f\rangle = \langle \upuparrows f\rangle^*$. □

PROPOSITION 1839. $\Lambda \upuparrows g = \upuparrows \Lambda g$ for every prestaroid g on boolean lattices.