THEOREM 1828. Let μ and ν be indexed (by some index set n) families of endofuncoids, and $f_i \in \mathsf{FCD}(\operatorname{Ob}\mu_i, \operatorname{Ob}\nu_i)$ for every $i \in n$. Then:

1°. $\forall i \in n : f_i \in \mathrm{C}(\mu_i, \nu_i) \Rightarrow \prod^{(A)} f \in \mathrm{C}\left(\prod^{(A)} \mu, \prod^{(A)} \nu\right)$;
2°. $\forall i \in n : f_i \in \mathrm{C}'(\mu_i, \nu_i) \Rightarrow \prod^{(A)} f \in \mathrm{C}'\left(\prod^{(A)} \mu, \prod^{(A)} \nu\right)$;
3°. $\forall i \in n : f_i \in \mathrm{C}''(\mu_i, \nu_i) \Rightarrow \prod^{(A)} f \in \mathrm{C}''\left(\prod^{(A)} \mu, \prod^{(A)} \nu\right)$.

PROOF. Similar to the previous theorem. □

THEOREM 1829. Let μ and ν be indexed (by some index set n) families of point-free endofuncoids between posets with least elements, and $f_i \in \mathsf{pFCD}(\operatorname{Ob}\mu_i, \operatorname{Ob}\nu_i)$ for every $i \in n$. Then:

1°. $\forall i \in n : f_i \in \mathrm{C}(\mu_i, \nu_i) \Rightarrow \prod^{(S)} f \in \mathrm{C}\left(\prod^{(S)} \mu, \prod^{(S)} \nu\right)$;
2°. $\forall i \in n : f_i \in \mathrm{C}'(\mu_i, \nu_i) \Rightarrow \prod^{(S)} f \in \mathrm{C}'\left(\prod^{(S)} \mu, \prod^{(S)} \nu\right)$;
3°. $\forall i \in n : f_i \in \mathrm{C}''(\mu_i, \nu_i) \Rightarrow \prod^{(S)} f \in \mathrm{C}''\left(\prod^{(S)} \mu, \prod^{(S)} \nu\right)$.

PROOF. Similar to the previous theorem. □

21.17. Upgrading and downgrading multifuncoids

LEMMA 1830. $\left\{ \frac{\langle f \rangle_k^* X}{X \in \operatorname{up} \prod_{i \in n \setminus \{k\}}^{\mathfrak{Z}_i} \mathcal{X}} \right\}$ is a filter base on $\mathfrak{A}_k$ for every family $(\mathfrak{A}_i, \mathfrak{Z}_i)$ of primary filtrators where $i \in n$ for some index set n (provided that f is a multifuncoid of the form $\mathfrak{Z}$ and $k \in n$ and $\mathcal{X} \in \prod_{i \in n \setminus \{k\}} \mathfrak{A}_i$).

PROOF. Let $\mathcal{K}, \mathcal{L} \in \left\{ \frac{\langle f \rangle_k^* X}{X \in \operatorname{up} \mathcal{X}} \right\}$. Then there exist $X, Y \in \operatorname{up} \mathcal{X}$ such that $\mathcal{K} = \langle f \rangle_k^* X$, $\mathcal{L} = \langle f \rangle_k^* Y$. We can take $Z \in \operatorname{up} \mathcal{X}$ such that $Z \sqsubseteq X, Y$. Then evidently $\langle f \rangle_k^* Z \sqsubseteq \mathcal{K}$ and $\langle f \rangle_k^* Z \sqsubseteq \mathcal{L}$ and $\langle f \rangle_k^* Z \in \left\{ \frac{\langle f \rangle_k^* X}{X \in \operatorname{up} \mathcal{X}} \right\}$. □

DEFINITION 1831. *Square* mult is a mult whose base and core are the same.

DEFINITION 1832. $\mathcal{L} \in [f] \Leftrightarrow \forall L \in \operatorname{up} \mathcal{L} : L \in [f]^*$ for every mult f.

DEFINITION 1833. $\langle f \rangle \mathcal{X} = \prod_{X \in \operatorname{up} \mathcal{X}} \langle f \rangle^* X$ for every mult f whose base is a complete lattice.

DEFINITION 1834. Let f be a mult whose base is a complete lattice. *Upgrading* of this mult is square mult $\upuparrows f$ with base $\upuparrows f = \operatorname{core} \upuparrows f = \operatorname{base} f$ and $\langle \upuparrows f \rangle^* \mathcal{X} = \langle f \rangle \mathcal{X}$ for every $\mathcal{X} \in \prod \operatorname{base} f$.

LEMMA 1835. $\mathcal{L}_i \not\asymp \langle \upuparrows f \rangle^* \mathcal{L}|_{(\operatorname{dom} \mathcal{L}) \setminus \{i\}} \Leftrightarrow \forall L \in \operatorname{up} \mathcal{L} : L_i \not\asymp \langle f \rangle^* L|_{(\operatorname{dom} \mathcal{L}) \setminus \{i\}}$, if every $((\operatorname{base} f)_i, (\operatorname{core} f)_i)$ is a primary filtrator over a meet-semilattice with least element.