

PROOF.

$$\begin{aligned} \uparrow^{\text{FCD}} A \left[f \times^{(C)} g \right] \uparrow^{\text{FCD}} B &\Leftrightarrow \\ \exists a \in \text{atoms } \uparrow^{\text{FCD}} A, b \in \text{atoms } \uparrow^{\text{FCD}} B : a \left[f \times^{(C)} g \right] b &\Leftrightarrow \\ \exists a \in \text{atoms } \uparrow^{\text{FCD}} A, b \in \text{atoms } \uparrow^{\text{FCD}} B : (\text{dom } a [f] \text{ dom } b \wedge \text{im } a [g] \text{ im } b) &\Rightarrow \\ \exists a_0 \in \text{atoms dom } \uparrow^{\text{FCD}} A, a_1 \in \text{atoms im } \uparrow^{\text{FCD}} A, & \\ b_0 \in \text{atoms dom } \uparrow^{\text{FCD}} B, b_1 \in \text{atoms im } \uparrow^{\text{FCD}} B : (a_0 [f] b_0 \wedge a_1 [g] b_1). & \end{aligned}$$

On the other hand:

$$\begin{aligned} \exists a_0 \in \text{atoms dom } \uparrow^{\text{FCD}} A, a_1 \in \text{atoms im } \uparrow^{\text{FCD}} A, & \\ b_0 \in \text{atoms dom } \uparrow^{\text{FCD}} B, b_1 \in \text{atoms im } \uparrow^{\text{FCD}} B : (a_0 [f] b_0 \wedge a_1 [g] b_1) &\Rightarrow \\ \exists a_0 \in \text{atoms dom } \uparrow^{\text{FCD}} A, a_1 \in \text{atoms im } \uparrow^{\text{FCD}} A, & \\ b_0 \in \text{atoms dom } \uparrow^{\text{FCD}} B, b_1 \in \text{atoms im } \uparrow^{\text{FCD}} B : (a_0 \times^{\text{FCD}} b_0 \not\neq f \wedge a_1 \times^{\text{FCD}} b_1 \not\neq g) &\Rightarrow \\ \exists a \in \text{atoms } \uparrow^{\text{FCD}} A, b \in \text{atoms } \uparrow^{\text{FCD}} B : (\text{dom } a [f] \text{ dom } b \wedge \text{im } a [g] \text{ im } b). & \end{aligned}$$

Also using the lemma we have

$$\begin{aligned} \exists a \in \text{atoms } \uparrow^{\text{FCD}} A, b \in \text{atoms } \uparrow^{\text{FCD}} B : (\text{dom } a [f] \text{ dom } b \wedge \text{im } a [g] \text{ im } b) &\Leftrightarrow \\ \exists a \in \text{atoms } \uparrow^{\text{RLD}} A, b \in \text{atoms } \uparrow^{\text{RLD}} B : (\text{dom } a [f] \text{ dom } b \wedge \text{im } a [g] \text{ im } b). & \end{aligned}$$

So

$$\begin{aligned} \uparrow^{\text{FCD}} A \left[f \times^{(C)} g \right] \uparrow^{\text{FCD}} B &\Leftrightarrow \\ \exists a \in \text{atoms } \uparrow^{\text{RLD}} A, b \in \text{atoms } \uparrow^{\text{RLD}} B : (\text{dom } a [f] \text{ dom } b \wedge \text{im } a [g] \text{ im } b) &\Leftrightarrow \\ \exists a \in \text{atoms } \uparrow^{\text{RLD}} A, b \in \text{atoms } \uparrow^{\text{RLD}} B : a \left[f \times^{(A)} g \right] b &\Leftrightarrow \\ \uparrow^{\text{RLD}} A \left[f \times^{(A)} g \right] \uparrow^{\text{RLD}} B. & \end{aligned}$$

□

COROLLARY 1815. $f \times^{(A)} g = \uparrow\downarrow (f \times^{(C)} g)$ where downgrading is taken on the filtrator

$$\left(\text{pFCD}(\text{FCD}(\text{Src } \circ f), \text{FCD}(\text{Dst } \circ f)), \text{FCD} \left(\mathcal{P} \prod (\text{Src } \circ f), \mathcal{P} \prod (\text{Dst } \circ f) \right) \right)$$

and upgrading is taken on the filtrator

$$\left(\text{pFCD}(\text{RLD}(\text{Src } \circ f), \text{RLD}(\text{Dst } \circ f)), \text{FCD} \left(\mathcal{P} \prod (\text{Src } \circ f), \mathcal{P} \prod (\text{Dst } \circ f) \right) \right).$$

where we equate n -ary relations with corresponding principal multifuncoids and principal multireloids, when appropriate.

PROOF. Leave as an exercise for the reader. □

CONJECTURE 1816. $\uparrow^{\text{FCD}} A \left[\prod^{(C)} f \right] \uparrow^{\text{FCD}} B \Leftrightarrow \uparrow^{\text{RLD}} A \left[\prod^{(A)} f \right] \uparrow^{\text{RLD}} B$ for every indexed family f of funcoids and $A \in \mathcal{P} \prod_{i \in \text{dom } f} \text{Src } f_i$, $B \in \mathcal{P} \prod_{i \in \text{dom } f} \text{Dst } f_i$.

THEOREM 1817. For every filters a_0, a_1, b_0, b_1 we have

$$a_0 \times^{\text{FCD}} b_0 \left[f \times^{(C)} g \right] a_1 \times^{\text{FCD}} b_1 \Leftrightarrow a_0 \times^{\text{RLD}} b_0 \left[f \times^{(A)} g \right] a_1 \times^{\text{RLD}} b_1.$$