

PROOF.

$$\begin{aligned}
& a \left[f \times^{(C)} g \right] b \Leftrightarrow \\
& a \circ f^{-1} \not\neq g^{-1} \circ b \Leftrightarrow \\
& (\text{dom } a \times^{\text{FCD}} \text{im } a) \circ f^{-1} \not\neq g^{-1} \circ (\text{dom } b \times^{\text{FCD}} \text{im } b) \Leftrightarrow \\
& \langle f \rangle \text{dom } a \times^{\text{FCD}} \text{im } a \not\neq \text{dom } b \times^{\text{FCD}} \langle g^{-1} \rangle \text{im } b \Leftrightarrow \\
& \langle f \rangle \text{dom } a \not\neq \text{dom } b \wedge \text{im } a \not\neq \langle g^{-1} \rangle \text{im } b \Leftrightarrow \\
& \text{dom } a [f] \text{dom } b \wedge \text{im } a [g] \text{im } b.
\end{aligned}$$

□

PROPOSITION 1811. $\mathcal{X} \left[\prod^{(A)} f \right] \mathcal{Y} \Leftrightarrow \forall i \in \text{dom } f : \text{Pr}_i^{\text{RLD}} \mathcal{X} [f_i] \text{Pr}_i^{\text{RLD}} \mathcal{Y}$ for every indexed family f of funcoids and $\mathcal{X} \in \text{RLD}(\text{Src } \circ f)$, $\mathcal{Y} \in \text{RLD}(\text{Dst } \circ f)$.

PROOF.

$$\begin{aligned}
& \mathcal{X} \left[\prod^{(A)} f \right] \mathcal{Y} \Leftrightarrow \\
& \exists a \in \text{atoms } \mathcal{X}, b \in \text{atoms } \mathcal{Y} : a \left[\prod^{(A)} f \right] b \Leftrightarrow \\
& \exists a \in \text{atoms } \mathcal{X}, b \in \text{atoms } \mathcal{Y} \forall i \in \text{dom } f : \text{Pr}_i^{\text{RLD}} a [f_i] \text{Pr}_i^{\text{RLD}} b \Leftrightarrow \\
& \forall i \in \text{dom } f \exists x \in \text{atoms } \text{Pr}_i^{\text{RLD}} \mathcal{X}, y \in \text{atoms } \text{Pr}_i^{\text{RLD}} \mathcal{Y} : x_i [f_i] y_i \Leftrightarrow \\
& \forall i \in \text{dom } f : \text{Pr}_i^{\text{RLD}} \mathcal{X} [f_i] \text{Pr}_i^{\text{RLD}} \mathcal{Y}.
\end{aligned}$$

□

COROLLARY 1812. $\mathcal{X} [f \times^{(A)} g] \mathcal{Y} \Leftrightarrow \text{dom } \mathcal{X} [f] \text{dom } \mathcal{Y} \wedge \text{im } \mathcal{X} [g] \text{im } \mathcal{Y}$ for funcoids f, g and reloids $\mathcal{X} \in \text{RLD}(\text{Src } f, \text{Src } g)$, and $\mathcal{Y} \in \text{RLD}(\text{Dst } f, \text{Dst } g)$.

LEMMA 1813. For every $A \in \mathbf{Rel}(X, Y)$ (for every sets X, Y) we have:

$$\left\{ \frac{(\text{dom } a, \text{im } a)}{a \in \text{atoms } \uparrow^{\text{FCD}} A} \right\} = \left\{ \frac{(\text{dom } a, \text{im } a)}{a \in \text{atoms } \uparrow^{\text{RLD}} A} \right\}.$$

PROOF. Let $x \in \left\{ \frac{(\text{dom } a, \text{im } a)}{a \in \text{atoms } \uparrow^{\text{RLD}} A} \right\}$. Take $x_0 = \text{dom } a$ and $x_1 = \text{im } a$ where $a \in \text{atoms } \uparrow^{\text{RLD}} A$.

Then $x_0 = \text{dom}(\text{FCD})a$ and $x_1 = \text{im}(\text{FCD})a$ and obviously $(\text{FCD})a \in \text{atoms } \uparrow^{\text{FCD}} A$. So $x \in \left\{ \frac{(\text{dom } a, \text{im } a)}{a \in \text{atoms } \uparrow^{\text{FCD}} A} \right\}$.

Let now $x \in \left\{ \frac{(\text{dom } a, \text{im } a)}{a \in \text{atoms } \uparrow^{\text{FCD}} A} \right\}$. Take $x_0 = \text{dom } a$ and $x_1 = \text{im } a$ where $a \in \text{atoms } \uparrow^{\text{FCD}} A$.

$x_0 [\uparrow^{\text{FCD}} A] x_1 \Leftrightarrow x_0 [(\text{FCD}) \uparrow^{\text{RLD}} A] x_1 \Leftrightarrow x_0 \times^{\text{RLD}} x_1 \not\neq \uparrow^{\text{RLD}} A$. Thus there exists atomic reloid x' such that $x' \in \text{atoms } \uparrow^{\text{RLD}} A$ and $\text{dom } x' = x_0$, $\text{im } x' = x_1$.

$$\text{So } x \in \left\{ \frac{(\text{dom } a', \text{im } a')}{a' \in \text{atoms } \uparrow^{\text{RLD}} A} \right\}. \quad \square$$

THEOREM 1814. $\uparrow^{\text{FCD}} A [f \times^{(C)} g] \uparrow^{\text{FCD}} B \Leftrightarrow \uparrow^{\text{RLD}} A [f \times^{(A)} g] \uparrow^{\text{RLD}} B$ for funcoids f, g , and **Rld**-morphisms $A : \text{Src } f \rightarrow \text{Src } g$, and $B : \text{Dst } f \rightarrow \text{Dst } g$.