So the value of x can be restored from $\prod^{(C)} x$ by this formula. □

21.13.3. Subatomic product.

PROPOSITION 1798. Values x_i (for every $i \in \operatorname{dom} x$) can be restored from the value of $\prod^{(A)} x$ provided that x is an indexed family of non-zero funcoids.

PROOF. Fix $k \in \operatorname{dom} f$. Let for some filters $\mathcal{X}$ and $\mathcal{Y}$

$$a = \begin{cases} \top^{\mathscr{F}(\operatorname{Base}(x))} & \text{if } i \neq k; \\ \mathcal{X} & \text{if } i = k \end{cases} \quad \text{and} \quad b = \begin{cases} \top^{\mathscr{F}(\operatorname{Base}(y))} & \text{if } i \neq k; \\ \mathcal{Y} & \text{if } i = k. \end{cases}$$

Then $\mathcal{X}\ [x_k]\ \mathcal{Y} \Leftrightarrow a_k\ [x_k]\ b_k \Leftrightarrow \forall i \in \operatorname{dom} f : a_i\ [x_i]\ b_i \Leftrightarrow \prod^{\mathsf{RLD}} a \left[\prod^{(A)} x\right] \prod^{\mathsf{RLD}} b$. So we have restored x_k from $\prod^{(A)} x$. □

DEFINITION 1799. For every funcoid $f : \prod A \to \prod B$ (where A and B are indexed families of typed sets) consider the funcoid $\Pr^{(A)}_k f$ defined by the formula

$$X \left[\Pr^{(A)}_k f\right]^* Y \Leftrightarrow \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} A} \left(\begin{cases} \top^{\mathscr{F}(A_i)} & \text{if } i \neq k; \\ \uparrow^{A_i} X & \text{if } i = k \end{cases}\right) [f] \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} B} \left(\begin{cases} \top^{\mathscr{F}(B_i)} & \text{if } i \neq k; \\ \uparrow^{B_i} Y & \text{if } i = k \end{cases}\right).$$

PROPOSITION 1800. $\Pr^{(A)}_k f$ is really a funcoid.

PROOF. $\neg\left(\bot \left[\Pr^{(A)}_k f\right]^* Y\right)$ is obvious.

$$\begin{aligned}
& I \sqcup J \left[\Pr^{(A)}_k f\right]^* Y \Leftrightarrow \\
& \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} A} \left(\begin{cases} \top^{\mathscr{F}(A_i)} & \text{if } i \neq k; \\ \uparrow^{A_i} (I \sqcup J) & \text{if } i = k \end{cases}\right) [f] \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} B} \left(\begin{cases} \top^{\mathscr{F}(B_i)} & \text{if } i \neq k; \\ \uparrow^{B_i} Y & \text{if } i = k \end{cases}\right) \Leftrightarrow \\
& \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} A} \left(\begin{cases} \top^{\mathscr{F}(A_i)} & \text{if } i \neq k; \\ \uparrow^{A_i} I \sqcup \uparrow^{A_i} J & \text{if } i = k \end{cases}\right) [f] \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} B} \left(\begin{cases} \top^{\mathscr{F}(B_i)} & \text{if } i \neq k; \\ \uparrow^{B_i} Y & \text{if } i = k \end{cases}\right) \Leftrightarrow \\
& \left(\prod^{\mathsf{RLD}}_{i \in \operatorname{dom} A} \left(\begin{cases} \top^{\mathscr{F}(A_i)} & \text{if } i \neq k; \\ \uparrow^{A_i} I & \text{if } i = k \end{cases}\right) \sqcup \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} A} \left(\begin{cases} \top^{\mathscr{F}(A_i)} & \text{if } i \neq k; \\ \uparrow^{A_i} J & \text{if } i = k \end{cases}\right)\right) [f] \\
& \qquad \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} B} \left(\begin{cases} \top^{\mathscr{F}(B_i)} & \text{if } i \neq k; \\ \uparrow^{B_i} Y & \text{if } i = k \end{cases}\right) \Leftrightarrow \\
& \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} A} \left(\begin{cases} \top^{\mathscr{F}(A_i)} & \text{if } i \neq k; \\ \uparrow^{A_i} I & \text{if } i = k \end{cases}\right) [f] \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} B} \left(\begin{cases} \top^{\mathscr{F}(B_i)} & \text{if } i \neq k; \\ \uparrow^{B_i} Y & \text{if } i = k \end{cases}\right) \vee \\
& \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} A} \left(\begin{cases} \top^{\mathscr{F}(A_i)} & \text{if } i \neq k; \\ \uparrow^{A_i} J & \text{if } i = k \end{cases}\right) [f] \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} B} \left(\begin{cases} \top^{\mathscr{F}(B_i)} & \text{if } i \neq k; \\ \uparrow^{B_i} Y & \text{if } i = k \end{cases}\right) \Leftrightarrow \\
& I \left[\Pr^{(A)}_k f\right]^* Y \vee J \left[\Pr^{(A)}_k f\right]^* Y.
\end{aligned}$$

The rest follows from symmetry. □

PROPOSITION 1801. For every funcoid $f : \prod A \to \prod B$ (where A and B are indexed families of typed sets) the funcoid $\Pr^{(A)}_k f$ conforms to the formula

$$\mathcal{X} \left[\Pr^{(A)}_k f\right] \mathcal{Y} \Leftrightarrow \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} A} \left(\begin{cases} \top^{\mathscr{F}(A_i)} & \text{if } i \neq k; \\ \mathcal{X} & \text{if } i = k \end{cases}\right) [f] \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} B} \left(\begin{cases} \top^{\mathscr{F}(B_i)} & \text{if } i \neq k; \\ \mathcal{Y} & \text{if } i = k \end{cases}\right).$$