$$\prod^{\mathsf{RLD}} a \left[\prod^{(A)} f\right] \prod^{\mathsf{RLD}} b \Leftrightarrow$$

$$\exists x \in \operatorname{atoms} \prod^{\mathsf{RLD}} a, y \in \operatorname{atoms} \prod^{\mathsf{RLD}} b : x \left[\prod^{(A)} f\right] y \Leftrightarrow$$

$$\exists x \in \operatorname{atoms} \prod^{\mathsf{RLD}} a, y \in \operatorname{atoms} \prod^{\mathsf{RLD}} b \forall i \in \operatorname{dom} f : \left\langle \Pr_i \right\rangle^* x \left[f_i\right] \left\langle \Pr_i \right\rangle^* y \Leftrightarrow$$

$$\forall i \in \operatorname{dom} f \exists x \in \operatorname{atoms} a_i, y \in \operatorname{atoms} b_i : x \left[f_i\right] y \Leftrightarrow$$

$$\forall i \in \operatorname{dom} f : a_i \left[f_i\right] b_i.$$

□

THEOREM 1788. $\left\langle \prod^{(A)} f \right\rangle x = \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} f} \langle f_i \rangle \Pr^{\mathsf{RLD}}_i x$ for an indexed family f of funcoids and $x \in \operatorname{atoms}^{\mathsf{RLD}(\lambda i \in \operatorname{dom} f : \operatorname{Src} f_i)}$ for every $n \in \operatorname{dom} f$.

PROOF. For every ultrafilter $y \in \mathscr{F}\left(\prod_{i \in \operatorname{dom} f} \operatorname{Dst} f_i\right)$ we have:

$$y \not\asymp \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} f} \langle f_i \rangle \Pr^{\mathsf{RLD}}_i x \Leftrightarrow$$

$$\forall i \in \operatorname{dom} f : \Pr^{\mathsf{RLD}}_i y \not\asymp \langle f_i \rangle \Pr^{\mathsf{RLD}}_i x \Leftrightarrow$$

$$\forall i \in \operatorname{dom} f : \Pr^{\mathsf{RLD}}_i x \left[f_i\right] \Pr^{\mathsf{RLD}}_i y \Leftrightarrow$$

$$x \left[\prod^{(A)} f\right] y \Leftrightarrow$$

$$y \not\asymp \left\langle \prod^{(A)} f \right\rangle x.$$

Thus $\left\langle \prod^{(A)} f \right\rangle x = \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} f} \langle f_i \rangle \Pr^{\mathsf{RLD}}_i x$. □

COROLLARY 1789. $\langle f \times^{(A)} g \rangle x = \langle f \rangle (\operatorname{dom} x) \times^{\mathsf{RLD}} \langle g \rangle (\operatorname{im} x)$ for atomic x.

21.13. On products and projections

CONJECTURE 1790. For principal funcoids $\prod^{(C)}$and $\prod^{(A)}$ coincide with the conventional product of binary relations.

21.13.1. Staroidal product. Let f be a staroid, whose form components are boolean lattices.

DEFINITION 1791. *Staroidal projection* of a staroid f is the filter $\Pr^{\mathsf{Strd}}_k f$ corresponding to the free star

$$(\operatorname{val} f)_k (\lambda i \in (\operatorname{arity} f) \setminus \{k\} : \top^{(\operatorname{form} f)_i}).$$

PROPOSITION 1792. $\Pr_k \operatorname{GR} \prod^{\mathsf{Strd}} x = \star x_k$ if x is an indexed family of proper filters, and $k \in \operatorname{dom} x$.