PROOF. To prove that $\prod^{(A)} f$ exists we need to prove (for every $a \in \operatorname{atoms}^{\mathsf{RLD}(\lambda i \in \operatorname{dom} f : \operatorname{Src} f_i)}$, $b \in \operatorname{atoms}^{\mathsf{RLD}(\lambda i \in \operatorname{dom} f : \operatorname{Dst} f_i)}$)

$$\forall X \in \operatorname{GR} a, Y \in \operatorname{GR} b$$
$$\exists x \in \operatorname{atoms} \uparrow^{\mathsf{RLD}(\lambda i \in \operatorname{dom} f : \operatorname{Src} f_i)} X, y \in \operatorname{atoms} \uparrow^{\mathsf{RLD}(\lambda i \in \operatorname{dom} f : \operatorname{Dst} f_i)} Y : x \left[\prod^{(A)} f \right] y \Rightarrow a \left[\prod^{(A)} f \right] b.$$

Let

$$\forall X \in \operatorname{GR} a, Y \in \operatorname{GR} b$$
$$\exists x \in \operatorname{atoms} \uparrow^{\mathsf{RLD}(\lambda i \in \operatorname{dom} f : \operatorname{Src} f_i)} X, y \in \operatorname{atoms} \uparrow^{\mathsf{RLD}(\lambda i \in \operatorname{dom} f : \operatorname{Dst} f_i)} Y : x \left[\prod^{(A)} f \right] y.$$

Then

$$\forall X \in \operatorname{GR} a, Y \in \operatorname{GR} b$$
$$\exists x \in \operatorname{atoms} \uparrow^{\mathsf{RLD}(\lambda i \in \operatorname{dom} f : \operatorname{Src} f_i)} X, y \in \operatorname{atoms} \uparrow^{\mathsf{RLD}(\lambda i \in \operatorname{dom} f : \operatorname{Dst} f_i)} Y$$
$$\forall i \in \operatorname{dom} f : \operatorname*{Pr}^{\mathsf{RLD}}_i x \, [f_i] \operatorname*{Pr}^{\mathsf{RLD}}_i y.$$

Then because $\operatorname{Pr}^{\mathsf{RLD}}_i x \in \operatorname{atoms} \uparrow^{\operatorname{Src} f_i} \operatorname{Pr}_i X$ and likewise for y:

$$\forall X \in \operatorname{GR} a, Y \in \operatorname{GR} b \forall i \in \operatorname{dom} f$$
$$\exists x \in \operatorname{atoms} \uparrow^{\operatorname{Src} f_i} \operatorname*{Pr}_i X, y \in \operatorname{atoms} \uparrow^{\operatorname{Dst} f_i} \operatorname*{Pr}_i Y : x \, [f_i] \, y.$$

Thus $\forall X \in \operatorname{GR} a, Y \in \operatorname{GR} b \forall i \in \operatorname{dom} f : \uparrow^{\operatorname{Src} f_i} \operatorname{Pr}_i X \, [f_i] \uparrow^{\operatorname{Dst} f_i} \operatorname{Pr}_i Y$;

$\forall X \in \operatorname{GR} a, Y \in \operatorname{GR} b \forall i \in \operatorname{dom} f : \operatorname{Pr}_i X \, [f_i]^* \operatorname{Pr}_i Y$.

Then $\forall X \in \langle \operatorname{Pr}_i \rangle^* \operatorname{GR} a, Y \in \langle \operatorname{Pr}_i \rangle^* \operatorname{GR} b : X \, [f_i]^* \, Y$.

Thus $\operatorname{Pr}^{\mathsf{RLD}}_i a \, [f_i] \operatorname{Pr}^{\mathsf{RLD}}_i b$. So

$$\forall i \in \operatorname{dom} f : \operatorname*{Pr}^{\mathsf{RLD}}_i a \, [f_i] \operatorname*{Pr}^{\mathsf{RLD}}_i b$$

and thus $a \left[\prod^{(A)} f \right] b$. □

REMARK 1784. It seems that the proof of the above theorem can be simplified using cross-composition product.

THEOREM 1785. $\prod^{(A)}_{i \in n} (g_i \circ f_i) = \prod^{(A)} g \circ \prod^{(A)} f$ for indexed (by an index set n) families f and g of funcoids such that $\forall i \in n : \operatorname{Dst} f_i = \operatorname{Src} g_i$.