Obvious 1774. It is really a filter base.

Obvious 1775. $\prod^{\mathsf{RLD}*} a \sqsupseteq \prod^{\mathsf{RLD}} a$.

Proposition 1776. $\prod^{\mathsf{RLD}*} a = \prod^{\mathsf{RLD}} a$ if n is finite.

Proof. Take $m = n$ to show that $\prod^{\mathsf{RLD}*} a \sqsubseteq \prod^{\mathsf{RLD}} a$. □

Proposition 1777. $\prod^{\mathsf{RLD}*} a = \bot^{\mathsf{RLD}(\lambda i \in n : \mathrm{Base}(a_i))}$ if a_i is the non-proper filter for some $i \in n$.

Proof. Take $A_i = \bot$ and $m = \{i\}$. Then $\prod_{i \in n} \left(\begin{cases} A_i & \text{if } i \in m \\ \mathrm{Base}(a_i) & \text{if } i \in n \setminus m \end{cases} \right) = \bot$. □

Example 1778. There exists an indexed family a of principal filters such that $\prod^{\mathsf{RLD}*} a$ is non-principal.

Proof. Let n be infinite and $\mathrm{Base}(a_i)$ is a set of at least two elements. Let each a_i be a trivial ultrafilter.

Every $\prod_{i \in n} \left(\begin{cases} A_i & \text{if } i \in m \\ \mathrm{Base}(a_i) & \text{if } i \in n \setminus m \end{cases} \right)$ has at least 2^n elements.

There are elements up $\prod^{\mathsf{RLD}} a$ with cardinality 1. They can't be elements of up $\prod^{\mathsf{RLD}*} a$ because of cardinality issues. □

Corollary 1779. There exists an indexed family a of principal filters such that $\prod^{\mathsf{RLD}*} a \neq \prod^{\mathsf{RLD}} a$.

Proof. Because $\prod^{\mathsf{RLD}} a$ is principal. □

Proposition 1780. $\Pr^{\mathsf{RLD}}_k \prod^{\mathsf{RLD}*} x = x_k$ for every indexed family x of proper filters.

Proof. $\Pr^{\mathsf{RLD}}_k \prod^{\mathsf{RLD}*} x = \langle \Pr_k \rangle^* \mathrm{GR} \prod^{\mathsf{RLD}*} x = x_k$. □

Theorem 1781. $\Pr^{\mathsf{RLD}}_i f \sqsubseteq \mathcal{A}_i$ for all $i \in n$ iff $f \sqsubseteq \prod^{\mathsf{RLD}*} \mathcal{A}$ (for every reloid f of arity n and n-indexed family $\mathcal{A}$ of filters on sets).

Proof. $f \sqsubseteq \prod^{\mathsf{RLD}*} \mathcal{A} \Rightarrow \Pr^{\mathsf{RLD}}_i f \sqsubseteq \Pr^{\mathsf{RLD}}_i \prod^{\mathsf{RLD}*} \mathcal{A} \sqsubseteq \mathcal{A}_i$.

Let now $\Pr^{\mathsf{RLD}}_i f \sqsubseteq \mathcal{A}_i$.

$f \sqsubseteq \prod \left(\begin{cases} \Pr^{\mathsf{RLD}}_i f & \text{if } i \in m \\ \mathrm{Base}(\mathrm{form}\, f)_i & \text{if } i \notin m \end{cases} \right)$ for finite $m \subseteq n$, as it can be easily be proved by induction.

It follows $f \sqsubseteq \prod^{\mathsf{RLD}*} \mathcal{A}$. □

21.12. Subatomic product of funcoids

Definition 1782. Let f be an indexed family of funcoids. Then $\prod^{(A)} f$ (*subatomic product*) is a funcoid $\prod_{i \in \mathrm{dom}\, f} \mathrm{Src}\, f_i \to \prod_{i \in \mathrm{dom}\, f} \mathrm{Dst}\, f_i$ such that for every $a \in \mathrm{atoms}^{\mathsf{RLD}(\lambda i \in \mathrm{dom}\, f : \mathrm{Src}\, f_i)}$, $b \in \mathrm{atoms}^{\mathsf{RLD}(\lambda i \in \mathrm{dom}\, f : \mathrm{Dst}\, f_i)}$

$$a \left[\prod^{(A)} f \right] b \Leftrightarrow \forall i \in \mathrm{dom}\, f : \Pr^{\mathsf{RLD}}_i a \, [f_i] \, \Pr^{\mathsf{RLD}}_i b.$$

Proposition 1783. The funcoid $\prod^{(A)} f$ exists.