thus $\prod_{a\in S}\prod^{\mathsf{RLD}} a = \prod^{\mathsf{RLD}}_{i\in\operatorname{dom}\mathfrak{Z}} \bigsqcap^{\mathscr{F}(\mathfrak{Z}_i)} \Pr_i S$.

Let $S \neq \emptyset$.

$\bigsqcap^{\mathscr{F}(\mathfrak{Z}_i)} \Pr_i S \sqsubseteq \bigsqcap^{\mathscr{F}(\mathfrak{Z}_i)}\{a_i\} = a_i$ for every $a \in S$ because $a_i \in \Pr_i S$. Thus $\prod^{\mathsf{RLD}}_{i\in\operatorname{dom}\mathfrak{Z}} \bigsqcap^{\mathscr{F}(\mathfrak{Z}_i)} \Pr_i S \sqsubseteq \prod^{\mathsf{RLD}} a$;

$$\bigsqcap_{a\in S}\prod^{\mathsf{RLD}} a \sqsupseteq \prod^{\mathsf{RLD}}_{i\in\operatorname{dom}\mathfrak{Z}} \bigsqcap^{\mathscr{F}(\mathfrak{Z}_i)} \Pr_i S.$$

Now suppose $F \in \operatorname{GR}\prod^{\mathsf{RLD}}_{i\in\operatorname{dom}\mathfrak{Z}} \bigsqcap^{\mathscr{F}(\mathfrak{Z}_i)} \Pr_i S$. Then there exists $X \in \operatorname{up}\prod_{i\in\operatorname{dom}\mathfrak{Z}} \bigsqcap^{\mathscr{F}(\mathfrak{Z}_i)} \Pr_i S$ such that $F \supseteq \prod X$. It is enough to prove that there exist $a \in S$ such that $F \in \operatorname{GR}\prod^{\mathsf{RLD}} a$. For this it is enough $\prod X \in \operatorname{GR}\prod^{\mathsf{RLD}} a$.

Really, $X_i \in \operatorname{up}\bigsqcap^{\mathscr{F}(\mathfrak{Z}_i)} \Pr_i S$ thus $X_i \in \operatorname{up} a_i$ for every $A \in S$ because $\Pr_i S \supseteq \{a_i\}$.

Thus $\prod X \in \operatorname{GR}\prod^{\mathsf{RLD}} a$. □

Definition 1770. I call a multireloid *principal* iff its graph is a principal filter.

Definition 1771. I call a multireloid *convex* iff it is a join of reloidal products.

Theorem 1772. $\operatorname{StarComp}(a \sqcup b, f) = \operatorname{StarComp}(a, f) \sqcup \operatorname{StarComp}(b, f)$ for multireloids a, b and an indexed family f of reloids with $\operatorname{Src} f_i = (\operatorname{form} a)_i = (\operatorname{form} b)_i$.

Proof.

$$\begin{aligned}
&\operatorname{GR}(\operatorname{StarComp}(a, f) \sqcup \operatorname{StarComp}(b, f)) = \\
&\bigsqcap\left\{\frac{\uparrow^{\mathsf{RLD}(\operatorname{form} a)} \operatorname{StarComp}(A, F)}{A \in \operatorname{GR} a, F \in \prod_{i\in n}\operatorname{GR} f_i}\right\} \sqcup \bigsqcap\left\{\frac{\uparrow^{\mathsf{RLD}(\operatorname{form} b)} \operatorname{StarComp}(B, F)}{B \in \operatorname{GR} b, F \in \prod_{i\in n}\operatorname{GR} f_i}\right\} = \\
&\bigsqcap\left\{\frac{\uparrow^{\mathsf{RLD}(\operatorname{form} a)} \operatorname{StarComp}(A, F) \sqcup \uparrow^{\mathsf{RLD}(\operatorname{form} b)} \operatorname{StarComp}(B, F)}{A \in \operatorname{GR} a, B \in \operatorname{GR} b, F \in \prod_{i\in n}\operatorname{GR} f_i}\right\} = \\
&\bigsqcap\left\{\frac{\uparrow^{\mathsf{RLD}(\operatorname{form} a)} (\operatorname{StarComp}(A, F) \cup \operatorname{StarComp}(B, F))}{A \in \operatorname{GR} a, B \in \operatorname{GR} b, F \in \prod_{i\in n}\operatorname{GR} f_i}\right\} = \\
&\bigsqcap\left\{\frac{\uparrow^{\mathsf{RLD}(\operatorname{form} a)} \operatorname{StarComp}(A \cup B, F)}{A \in \operatorname{GR} a, B \in \operatorname{GR} b, F \in \prod_{i\in n}\operatorname{GR} f_i}\right\} = \\
&\bigsqcap\left\{\frac{\uparrow^{\mathsf{RLD}(\operatorname{form} a)} \operatorname{StarComp}(C, F)}{C \in \operatorname{GR}(a \sqcup b), F \in \prod_{i\in n}\operatorname{GR} f_i}\right\} = \\
&\operatorname{GR}\operatorname{StarComp}(a \sqcup b, f).
\end{aligned}$$

□

21.11.1. Starred reloidal product. Tychonoff product of topological spaces inspired me the following definition, which seems possibly useful just like Tychonoff product:

Definition 1773. Let a be an n-indexed (n is an arbitrary index set) family of filters on sets. $\prod^{\mathsf{RLD}*} a$ (*starred reloidal product*) is the reloid of the form $\prod_{i\in n}\operatorname{Base}(a_i)$ induced by the filter base

$$\left\{\frac{\prod_{i\in n}\left(\begin{cases} A_i & \text{if } i \in m \\ \operatorname{Base}(a_i) & \text{if } i \in n \setminus m\end{cases}\right)}{m \text{ is a finite subset of } n, A \in \prod(a|_m)}\right\}.$$