CONJECTURE 1766. $\operatorname{GR}\operatorname{StarComp}(a \sqcup b, f) = \operatorname{GR}\operatorname{StarComp}(a, f) \sqcup \operatorname{GR}\operatorname{StarComp}(b, f)$ if f is a reloid and a, b are multireloids of the same form, composable with f.

THEOREM 1767. $\prod^{\mathsf{RLD}} A = \bigsqcup\left\{ \frac{\prod^{\mathsf{RLD}} a}{a \in \prod_{i \in \operatorname{dom} A} \operatorname{atoms} A_i} \right\}$ for every indexed family A of filters on powersets.

PROOF. Obviously $\prod^{\mathsf{RLD}} A \sqsupseteq \bigsqcup\left\{ \frac{\prod^{\mathsf{RLD}} a}{a \in \prod_{i \in \operatorname{dom} A} \operatorname{atoms} A_i} \right\}$.

Reversely, let $K \in \operatorname{GR} \bigsqcup\left\{ \frac{\prod^{\mathsf{RLD}} a}{a \in \prod_{i \in \operatorname{dom} A} \operatorname{atoms} A_i} \right\}$.

Consequently $K \in \operatorname{GR} \prod^{\mathsf{RLD}} a$ for every $a \in \prod_{i \in \operatorname{dom} A} \operatorname{atoms} A_i$; $K \supseteq \prod X$ and thus $K \supseteq \bigcup_{a \in \prod_{i \in \operatorname{dom} a} \operatorname{atoms} A_i} \prod X_a$ for some $X_a \in \prod_{i \in \operatorname{dom} a} \operatorname{atoms} A_i$.

But $\bigcup_{a \in \prod_{i \in \operatorname{dom} a} \operatorname{atoms} A_i} \prod X_a = \prod_{i \in \operatorname{dom} A} \bigcup_{a \in \operatorname{atoms} A_i} \langle \Pr_i \rangle^* X_a \supseteq \prod_{j \in \operatorname{dom} A} Z_j$ for some $Z_j \in \operatorname{up} A_j$ because $\langle \Pr_i \rangle^* X \in \operatorname{up} a_i$ and our lattice is atomistic. So $K \in \operatorname{GR} \prod^{\mathsf{RLD}} A$. □

THEOREM 1768. Let a, b be indexed families of filters on powersets of the same form $\mathfrak{A}$. Then

$$\prod^{\mathsf{RLD}} a \sqcap \prod^{\mathsf{RLD}} b = \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} \mathfrak{A}} (a_i \sqcap b_i).$$

PROOF.

$$\begin{aligned}
&\operatorname{up}\left(\prod^{\mathsf{RLD}} a \sqcap \prod^{\mathsf{RLD}} b \right) = \\
&\bigsqcap^{\mathsf{RLD}(\mathfrak{A})} \left\{ \frac{P \cap Q}{P \in \operatorname{GR} \prod^{\mathsf{RLD}} a, Q \in \prod^{\mathsf{RLD}} b} \right\} = \\
&\bigsqcap^{\mathsf{RLD}(\mathfrak{A})} \left\{ \frac{\prod p \cap \prod q}{p \in \operatorname{up} \prod a, q \in \operatorname{up} \prod b} \right\} = \\
&\bigsqcap^{\mathsf{RLD}(\mathfrak{A})} \left\{ \frac{\prod_{i \in \operatorname{dom} \mathfrak{A}} (p_i \cap q_i)}{p \in \prod \operatorname{up} a, q \in \prod \operatorname{up} b} \right\} = \\
&\bigsqcap^{\mathsf{RLD}(\mathfrak{A})} \left\{ \frac{\prod r}{r \in \operatorname{up} \prod_{i \in \operatorname{dom} \mathfrak{A}} (a_i \sqcap b_i)} \right\} = \\
&\operatorname{up} \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} \mathfrak{A}} (a_i \sqcap b_i).
\end{aligned}$$

□

THEOREM 1769. If $S \in \mathscr{P} \prod_{i \in \operatorname{dom} \mathfrak{Z}} \mathscr{F}(\mathfrak{Z}_i)$ where $\mathfrak{Z}$ is an indexed family of sets, then

$$\bigsqcap_{a \in S} \prod^{\mathsf{RLD}} a = \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} \mathfrak{Z}} \bigsqcap^{\mathscr{F}(\mathfrak{Z}_i)} \Pr_i S.$$

PROOF. If $S = \emptyset$ then $\bigsqcap_{a \in S} \prod^{\mathsf{RLD}} a = \bigsqcap \emptyset = \top^{\mathsf{RLD}(\mathfrak{Z})}$ and

$$\prod^{\mathsf{RLD}}_{i \in \operatorname{dom} \mathfrak{Z}} \bigsqcap^{\mathscr{F}(\mathfrak{Z}_i)} \Pr_i S = \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} \mathfrak{Z}} \bigsqcap^{\mathscr{F}(\mathfrak{Z}_i)} \emptyset = \prod^{\mathsf{RLD}}_{i \in \operatorname{dom} \mathfrak{Z}} \top^{\mathscr{F}(\mathfrak{Z}_i)} = \top^{\mathsf{RLD}(\mathfrak{Z})},$$