$L \in \operatorname{GR} \operatorname{StarComp}^{(a)}(a, f) \Leftrightarrow \exists y \in \operatorname{GR} a \cap \prod_{i\in n} \operatorname{atoms}^{\mathfrak{A}_i} : y\ R(f)\ L.$

$$\begin{aligned}
&L \in \operatorname{GR} \operatorname{StarComp}^{(a)}(\operatorname{StarComp}(a, f), g) \Leftrightarrow\\
&\exists p \in \operatorname{GR} \operatorname{StarComp}^{(a)}(a, f) \cap \prod_{i\in n} \operatorname{atoms}^{(\operatorname{Dst} f)_i} : p\ R(g)\ L \Leftrightarrow\\
&\exists p \in \prod_{i\in n} \operatorname{atoms}^{(\operatorname{Dst} f)_i}, y \in \operatorname{GR} a \cap \prod_{i\in n} \operatorname{atoms}^{(\operatorname{Src} f)_i} : (y\ R(f)\ p \wedge p\ R(g)\ L) \Leftrightarrow\\
&\exists y \in \operatorname{GR} a \cap \prod_{i\in n} \operatorname{atoms}^{(\operatorname{Src} f)_i} : y\ (R(g) \circ R(f))\ L \Leftrightarrow\\
&\exists y \in \operatorname{GR} a \cap \prod_{i\in n} \operatorname{atoms}^{(\operatorname{Src} f)_i} : y\ R(\lambda i \in n : g_i \circ f_i)\ L \Leftrightarrow\\
&\exists y \in \operatorname{GR} a \cap \prod_{i\in n} \operatorname{atoms}^{(\operatorname{Src} f)_i}\ y \forall i \in n : y_i\ [g_i \circ f_i]\ L_i \Leftrightarrow\\
&L \in \operatorname{GR} \operatorname{StarComp}^{(a)}(a, \lambda i \in n : g_i \circ f_i)
\end{aligned}$$

because $p \in \operatorname{GR} \operatorname{StarComp}^{(a)}(a, f) \Leftrightarrow \exists y \in \operatorname{GR} a \cap \prod_{i\in n} \operatorname{atoms}^{(\operatorname{Src} f)_i}\ y : y\ R(f)\ p$.

2°. It follows from the lemma above.

□

THEOREM 1749. $\left\langle \prod^{(a)} f \right\rangle \prod^{\mathsf{Strd}} a = \prod^{\mathsf{Strd}}_{i\in n} \langle f_i \rangle a_i$ for every family $f = f_{i\in n}$ of pointfree funcoids between atomic posets and $a = a_{i\in n}$ where $a_i \in \operatorname{Src} f_i$.

PROOF.

$$\begin{aligned}
&L \in \operatorname{GR} \left\langle \prod^{(a)} f \right\rangle \prod^{\mathsf{Strd}} a \Leftrightarrow\\
&L \in \operatorname{GR} \operatorname{StarComp}^{(a)} \left(\prod^{\mathsf{Strd}} a, f \right) \Leftrightarrow\\
&\exists y \in \prod_{i \in \operatorname{dom} \mathfrak{A}} \operatorname{atoms}^{\mathfrak{A}_i} \forall i \in n : (y_i\ [f_i]\ L_i \wedge y_i \not\asymp a_i) \Leftrightarrow\\
&\forall i \in n \exists y \in \operatorname{atoms}^{\mathfrak{A}_i} : (y\ [f_i]\ L_i \wedge y \not\asymp a_i) \Leftrightarrow\\
&\forall i \in n : a_i\ [f_i]\ L_i \Leftrightarrow\\
&\forall i \in n : L_i \not\asymp \langle f_i \rangle a_i \Leftrightarrow\\
&L \in \operatorname{GR} \prod^{\mathsf{Strd}}_{i\in n} \langle f_i \rangle a_i.
\end{aligned}$$

□

CONJECTURE 1750. $\operatorname{StarComp}^{(a)}(a \sqcup b, f) = \operatorname{StarComp}^{(a)}(a, f) \sqcup \operatorname{StarComp}^{(a)}(b, f)$ for anchored relations a, b of a form $\mathfrak{A}$, where every $\mathfrak{A}_i$ is a distributive lattice, and an indexed family f of pointfree funcoids with $\operatorname{Src} f_i = \mathfrak{A}_i$.

21.10.7. Simple product of pointfree funcoids.

DEFINITION 1751. Let f be an indexed family of pointfree funcoids with every $\operatorname{Src} f_i$ and $\operatorname{Dst} f_i$ (for all $i \in \operatorname{dom} f$) being a poset with least element. *Simple product* of f is

$$\prod^{(S)} f = \left(\lambda x \in \prod_{i\in \operatorname{dom} f} \operatorname{Src} f_i : \lambda i \in \operatorname{dom} f : \langle f_i \rangle x_i, \lambda y \in \prod_{i\in \operatorname{dom} f} \operatorname{Dst} f_i : \lambda i \in \operatorname{dom} f : \langle f_i^{-1} \rangle y_i \right).$$