1°. $(h\circ g)\circ f = \lambda i \in \operatorname{dom} f : (h_i \circ g_i)\circ f_i = \lambda i \in \operatorname{dom} f : h_i \circ (g_i \circ f_i) = h \circ (g \circ f)$;

$$g \circ (f \circ m) = \operatorname{StarComp}(\operatorname{StarComp}(m, f), g) = \operatorname{StarComp}(m, \lambda i \in \operatorname{arity} m : g_i \circ f_i) = \operatorname{StarComp}(m, g \circ f) = (g \circ f) \circ m;$$

$f \circ (m \circ 1_{\mathrm{None}}) = f \circ m = (f \circ m) \circ 1_{\mathrm{None}}$.

2°. $m \circ 1_{\mathrm{None}} = m$; $1_{\operatorname{Dst} m} \circ m = \operatorname{StarComp}(m, \lambda i \in \operatorname{arity} m : 1_{\mathrm{Obj}_m i}) = m$. □

Remark 1717. I call the above defined category *abrupt category* because (excluding identity morphisms) it allows composition with an $m \in M$ only on the left (not on the right) so that the morphism m is "abrupt" on the right.

By $[\![x_0, \ldots, x_{n-1}]\!]$ I denote an n-tuple.

Definition 1718. Precategory with star morphisms *induced* by a dagger precategory C is:

- The base category is C.
- Star-morphisms are morphisms of C.
- $\operatorname{arity} f = \{0, 1\}$.
- $\mathrm{Obj}_m = [\![\operatorname{Src} m, \operatorname{Dst} m]\!]$.
- $\operatorname{StarComp}(m, [\![f, g]\!]) = g \circ m \circ f^\dagger$.

Let prove it is really a precategory with star-morphisms.

Proof. We need to prove the associativity law:

$$\operatorname{StarComp}(\operatorname{StarComp}(m, [\![f, g]\!]), [\![p, q]\!]) = \operatorname{StarComp}(m, [\![p \circ f, q \circ g]\!]).$$

Really,

$$\operatorname{StarComp}(\operatorname{StarComp}(m, [\![f, g]\!]), [\![p, q]\!]) = \operatorname{StarComp}(g \circ m \circ f^\dagger, [\![p, q]\!]) = q \circ g \circ m \circ f^\dagger \circ p^\dagger = q \circ g \circ m \circ (p \circ f)^\dagger = \operatorname{StarComp}(m, [\![p \circ f, q \circ g]\!]).$$

□

Definition 1719. Category with star morphisms *induced* by a dagger category C is the above defined precategory with star-morphisms.

That it is a category (the law of composition with identity) is trivial.

Remark 1720. We can carry definitions (such as below defined cross-composition product) from categories with star-morphisms into plain dagger categories. This allows us to research properties of cross-composition product of indexed families of morphisms for categories with star-morphisms without separately considering the special case of dagger categories and just binary star-composition product.

21.9.1. Abrupt of quasi-invertible categories with star-morphisms.

Definition 1721. The abrupt partially ordered precategory of a partially ordered precategory with star-morphisms is the abrupt precategory with the following order of morphisms:

- Indexed (by $\operatorname{arity} m$ for some $m \in M$) families of morphisms of C are ordered as function spaces of posets.
- Star-morphisms (which are morphisms $\mathrm{None} \to \mathrm{Obj}_m$ for some $m \in M$) are ordered in the same order as in the precategory with star-morphisms.
- Morphisms $\mathrm{None} \to \mathrm{None}$ which are only the identity morphism ordered by the unique order on this one-element set.