1°. $\prod^{(\mathrm{ord})} F$ is a prestaroid if every F_i is a prestaroid.
2°. $\prod^{(\mathrm{ord})} F$ is a staroid if every F_i is a staroid.
3°. $\prod^{(\mathrm{ord})} F$ is a completary staroid if every F_i is a completary staroid.

PROOF. Use the fact that $\mathrm{GR}\prod^{(\mathrm{ord})} F = \left\{ \frac{F\circ\left(\bigoplus(\mathrm{dom}\circ F)\right)^{-1}}{F\in\mathrm{GR}\prod^{(D)} f} \right\}$. □

DEFINITION 1707. $f \times^{(\mathrm{ord})} g = \prod^{(\mathrm{ord})} [\![f, g]\!]$.

REMARK 1708. If f and g are binary funcoids, then $f \times^{(\mathrm{ord})} g$ is ternary.

21.9. Star categories

DEFINITION 1709. A *precategory with star-morphisms* consists of

1°. a precategory C (*the base precategory*);
2°. a set M (*star-morphisms*);
3°. a function "arity" defined on M (how many objects are connected by this star-morphism);
4°. a function $\mathrm{Obj}_m : \mathrm{arity}\, m \to \mathrm{Obj}(C)$ defined for every $m \in M$;
5°. a function (*star composition*) $(m, f) \mapsto \mathrm{StarComp}(m, f)$ defined for $m \in M$ and f being an $(\mathrm{arity}\, m)$-indexed family of morphisms of C such that $\forall i \in \mathrm{arity}\, m : \mathrm{Src}\, f_i = \mathrm{Obj}_m\, i$ ($\mathrm{Src}\, f_i$ is the source object of the morphism f_i) such that

such that it holds:

1°. $\mathrm{StarComp}(m, f) \in M$;
2°. $\mathrm{arity}\,\mathrm{StarComp}(m, f) = \mathrm{arity}\, m$;
3°. $\mathrm{Obj}_{\mathrm{StarComp}(m,f)}\, i = \mathrm{Dst}\, f_i$;
4°. (*associativity law*)

$$\mathrm{StarComp}(\mathrm{StarComp}(m, f), g) = \mathrm{StarComp}(m, \lambda i \in \mathrm{arity}\, m : g_i \circ f_i).$$

The meaning of the set M is an extension of C having as morphisms things with arbitrary (possibly infinite) indexed set Obj_m of objects, not just two objects as morphisms of C have only source and destination.

DEFINITION 1710. I will call Obj_m the *form* of the star-morphism m.

(Having fixed a precategory with star-morphisms) I will denote $\mathrm{StarHom}(P)$ the set of star-morphisms of the form P.

PROPOSITION 1711. The sets $\mathrm{StarHom}(P)$ are disjoint (for different P).

PROOF. If two star-morphisms have different forms, they are clearly not equal. □

DEFINITION 1712. A *category with star-morphisms* is a precategory with star-morphisms whose base is a category and the following equality (*the law of composition with identity*) holds for every star-morphism m:

$$\mathrm{StarComp}(m, \lambda i \in \mathrm{arity}\, m : 1_{\mathrm{Obj}_m\, i}) = m.$$

DEFINITION 1713. A *partially ordered precategory with star-morphisms* is a category with star-morphisms, whose base precategory is a partially ordered precategory and every set $\mathrm{StarHom}(X)$ is partially ordered for every X, such that:

$$m_0 \sqsubseteq m_1 \wedge f_0 \sqsubseteq f_1 \Rightarrow \mathrm{StarComp}(m_0, f_0) \sqsubseteq \mathrm{StarComp}(m_1, f_1)$$