2°. From the lemma.
3°. We need to prove

$$L_0 \sqcup L_1 \in \operatorname{GR} \prod^{(D)} F \Leftrightarrow$$
$$\exists c \in \{0,1\}^{\operatorname{arity} \prod^{(D)} F} : \left(\lambda i \in \operatorname{arity} \prod^{(D)} F : L_{c(i)} i\right) \in \operatorname{GR} \prod^{(D)} F$$

for every $L_0, L_1 \in \prod \operatorname{form} \prod^{(D)} F$ that is $L_0, L_1 \in \prod \operatorname{uncurry}(\operatorname{form} \circ F)$.

Really $L_0 \sqcup L_1 \in \operatorname{GR} \prod^{(D)} F \Leftrightarrow L_0 \sqcup L_1 \in \left\{ \frac{\operatorname{uncurry} z}{z \in \prod(\operatorname{GR} \circ F)} \right\}$.

$$\exists c \in \{0,1\}^{\operatorname{arity} \prod^{(D)} F} : \left(\lambda i \in \operatorname{arity} \prod^{(D)} F : L_{c(i)} i\right) \in \operatorname{GR} \prod^{(D)} F \Leftrightarrow$$
$$\exists c \in \{0,1\}^{\operatorname{arity} \prod^{(D)} F} : \left(\lambda i \in \operatorname{arity} \prod^{(D)} F : L_{c(i)} i\right) \in \left\{ \frac{\operatorname{uncurry} z}{z \in \prod(\operatorname{GR} \circ F)} \right\} \Leftrightarrow$$
$$\exists c \in \{0,1\}^{\operatorname{arity} \prod^{(D)} F} : \operatorname{curry}\left(\lambda i \in \operatorname{arity} \prod^{(D)} F : L_{c(i)} i\right) \in \prod(\operatorname{GR} \circ F) \Leftrightarrow$$
$$\exists c \in \{0,1\}^{\operatorname{arity} \prod^{(D)} F} : \operatorname{curry}\left(\lambda (i,j) \in \operatorname{arity} \prod^{(D)} F : L_{c(i,j)} (i,j)\right) \in \prod(\operatorname{GR} \circ F) \Leftrightarrow$$
$$\exists c \in \{0,1\}^{\operatorname{arity} \prod^{(D)} F} : (\lambda i \in \operatorname{dom} F : (\lambda j \in \operatorname{dom} F_i : L_{c(i,j)}(i,j))) \in \prod(\operatorname{GR} \circ F) \Leftrightarrow$$
$$\exists c \in \{0,1\}^{\operatorname{arity} \prod^{(D)} F} \forall i \in \operatorname{dom} F : (\lambda j \in \operatorname{dom} F_i : L_{c(i,j)}(i,j)) \in \operatorname{GR} F_i \Leftrightarrow$$
$$\forall i \in \operatorname{dom} F \exists c \in \{0,1\}^{\operatorname{dom} F_i} : (\lambda j \in \operatorname{dom} F_i : L_{c(j)}(i,j)) \in \operatorname{GR} F_i \Leftrightarrow$$
$$\forall i \in \operatorname{dom} F \exists c \in \{0,1\}^{\operatorname{dom} F_i} : (\lambda j \in \operatorname{dom} F_i : (\operatorname{curry}(L_{c(j)}) i) j) \in \operatorname{GR} F_i \Leftrightarrow$$
$$\forall i \in \operatorname{dom} F : \operatorname{curry}(L_0) i \sqcup \operatorname{curry}(L_1) i \in \operatorname{GR} F_i \Leftrightarrow$$
$$\forall i \in \operatorname{dom} F : (\operatorname{curry}(L_0) \sqcup \operatorname{curry}(L_1)) i \in \operatorname{GR} F_i \Leftrightarrow$$
$$\forall i \in \operatorname{dom} F : \operatorname{curry}(L_0 \sqcup L_1) i \in \operatorname{GR} F_i \Leftrightarrow$$
$$L_0 \sqcup L_1 \in \left\{ \frac{\operatorname{uncurry} z}{z \in \prod(\operatorname{GR} \circ F)} \right\} \Leftrightarrow$$
$$L_0 \sqcup L_1 \in \operatorname{GR} \prod^{(D)} F.$$

□

For staroids it is defined *ordinated product* $\prod^{(\mathrm{ord})}$ as defined in the section 3.7.4 above.

Obvious 1705. If f and g are anchored relations and there exists a bijection φ from $\operatorname{arity} g$ to $\operatorname{arity} f$ such that $\left\{ \frac{F \circ \varphi}{F \in \operatorname{GR} f} \right\} = \operatorname{GR} g$, then:

1°. f is a prestaroid iff g is a prestaroid.
2°. f is a staroid iff g is a staroid.
3°. f is a completary staroid iff g is a completary staroid.

Corollary 1706. Let F be an indexed family of anchored relations and every $(\operatorname{form} F)_i$ be a join-semilattice.