PROOF.

1°. Let $q \in \operatorname{arity}\prod^{(D)} F$ that is $q = (i, j)$ where $i \in \operatorname{dom} F$, $j \in \operatorname{arity} F_i$; let

$$L \in \prod\left(\left(\operatorname{form}\prod^{(D)} F\right)|_{\left(\operatorname{arity}\prod^{(D)} F\right)\setminus\{q\}}\right)$$

that is $L_{(i',j')} \in \left(\operatorname{form}\prod^{(D)} F\right)_{(i',j')}$ for every $(i',j') \in \left(\operatorname{arity}\prod^{(D)} F\right) \setminus \{q\}$, that is $L_{(i',j')} \in (\operatorname{form} F_{i'})_{j'}$. We have $X \in \left(\operatorname{form}\prod^{(D)} F\right)_{(i,j)} \Leftrightarrow X \in (\operatorname{form} F_i)_j$. So

$$\left(\operatorname{val}\prod^{(D)} F\right)_{(i,j)} L = \left\{\frac{X \in (\operatorname{form} F_i)_j}{L \cup \{((i,j), X)\} \in \operatorname{GR}\prod^{(D)} F}\right\} =$$

$$\left\{\frac{X \in (\operatorname{form} F_i)_j}{\exists z \in \prod(\operatorname{GR}\circ F) : L \cup \{((i,j), X)\} = \operatorname{uncurry} z}\right\} =$$

$$\left\{\frac{X \in (\operatorname{form} F_i)_j}{\exists z \in \prod\left((\operatorname{GR}\circ F)|_{\left(\operatorname{arity}\prod^{(D)} F\right)\setminus\{(i,j)\}}\right), v \in \operatorname{GR} F_i : (L = \operatorname{uncurry} z \wedge v_j = X)}\right\} =$$

$$\left\{\frac{X \in (\operatorname{form} F_i)_j}{\exists z \in \prod\left((\operatorname{GR}\circ F)|_{\left(\operatorname{arity}\prod^{(D)} F\right)\setminus\{(i,j)\}}\right) : L = \operatorname{uncurry} z \wedge \exists v \in \operatorname{GR} F_i : v_j = X}\right\}.$$

If $\exists z \in \prod\left((\operatorname{GR}\circ F)|_{\left(\operatorname{arity}\prod^{(D)} F\right)\setminus\{(i,j)\}}\right) : L = \operatorname{uncurry} z$ is false then $\left(\operatorname{val}\prod^{(D)} F\right)_{(i,j)} L = \emptyset$ is a free star. We can assume it is true. So

$$\left(\operatorname{val}\prod^{(D)} F\right)_{(i,j)} L = \left\{\frac{X \in (\operatorname{form} F_i)_j}{\exists v \in \operatorname{GR} F_i : v_j = X}\right\} =$$

$$\left\{\frac{X \in (\operatorname{form} F_i)_j}{\exists K \in (\operatorname{form} F_i)|_{(\operatorname{arity} F_i)\setminus\{j\}} : K \cup \{(j, X)\} \in \operatorname{GR} F_i}\right\} =$$

$$\left\{\frac{X \in (\operatorname{form} F_i)_j}{\exists K \in (\operatorname{form} F_i)|_{(\operatorname{arity} F_i)\setminus\{j\}} : X \in (\operatorname{val} F_i)_j K}\right\}.$$

Thus

$$A \sqcup B \in \left(\operatorname{val}\prod^{(D)} F\right)_{(i,j)} L \Leftrightarrow$$

$$\exists K \in (\operatorname{form} F_i)|_{(\operatorname{arity} F_i)\setminus\{j\}} : A \sqcup B \in (\operatorname{val} F_i)_j K \Leftrightarrow$$

$$\exists K \in (\operatorname{form} F_i)|_{(\operatorname{arity} F_i)\setminus\{j\}} : (A \in (\operatorname{val} F_i)_j K \vee B \in (\operatorname{val} F_i)_j K) \Leftrightarrow$$

$$\begin{array}{c}\exists K \in (\operatorname{form} F_i)|_{(\operatorname{arity} F_i)\setminus\{j\}} : A \in (\operatorname{val} F_i)_j K \vee \\ \exists K \in (\operatorname{form} F_i)|_{(\operatorname{arity} F_i)\setminus\{j\}} : B \in (\operatorname{val} F_i)_j K\end{array} \Leftrightarrow$$

$$A \in \left(\operatorname{val}\prod^{(D)} F\right)_{(i,j)} L \vee B \in \left(\operatorname{val}\prod^{(D)} F\right)_{(i,j)} L.$$

Least element $\bot$ is not in $\left(\operatorname{val}\prod^{(D)} F\right)_{(i,j)} L$ because $K \cup \{(j, \bot)\} \notin \operatorname{GR} F_i$.