Really,

$$\begin{aligned}
L_i \not\asymp \alpha_i L|_{(\operatorname{dom} L)\setminus\{i\}} &\Leftrightarrow \\
L_i \not\asymp \langle f_i\rangle^* L|_{(\operatorname{dom} L)\setminus\{i\}} \sqcup \langle g_i\rangle^* L|_{(\operatorname{dom} L)\setminus\{i\}} &\Leftrightarrow \\
L_i \not\asymp \langle f_i\rangle^* L|_{(\operatorname{dom} L)\setminus\{i\}} \vee L_i \not\asymp \langle g_i\rangle^* L|_{(\operatorname{dom} L)\setminus\{i\}} &\Leftrightarrow \\
L_j \not\asymp \langle f_j\rangle^* L|_{(\operatorname{dom} L)\setminus\{j\}} \vee L_j \not\asymp \langle g_j\rangle^* L|_{(\operatorname{dom} L)\setminus\{j\}} &\Leftrightarrow \\
L_j \not\asymp \langle f_j\rangle^* L|_{(\operatorname{dom} L)\setminus\{j\}} \sqcup \langle g_j\rangle^* L|_{(\operatorname{dom} L)\setminus\{j\}} &\Leftrightarrow \\
L_j \not\asymp \alpha_j L|_{(\operatorname{dom} L)\setminus\{j\}}.&
\end{aligned}$$

□

THEOREM 1682. $\bigsqcup^{\mathsf{pFCD}(\mathfrak{A})} F = \bigsqcup F$ for every set F of multifuncoids for the same indexed family of join infinite distributive complete lattices filtrators.

PROOF. $\alpha_i x \overset{\mathrm{def}}{=} \bigsqcup_{f\in F} \langle f\rangle_i^* x$. It is enough to prove that α is a multifuncoid.

We need to prove:

$$L_i \not\asymp \alpha_i L|_{(\operatorname{dom} L)\setminus\{i\}} \Leftrightarrow L_j \not\asymp \alpha_j L|_{(\operatorname{dom} L)\setminus\{j\}}.$$

Really,

$$\begin{aligned}
L_i \not\asymp \alpha_i L|_{(\operatorname{dom} L)\setminus\{i\}} &\Leftrightarrow \\
L_i \not\asymp \bigsqcup_{f\in F} \langle f_i\rangle^* L|_{(\operatorname{dom} L)\setminus\{i\}} &\Leftrightarrow \\
\exists f \in F : L_i \not\asymp \langle f_i\rangle^* L|_{(\operatorname{dom} L)\setminus\{i\}} &\Leftrightarrow \\
\exists f \in F : L_j \not\asymp \langle f_j\rangle^* L|_{(\operatorname{dom} L)\setminus\{j\}} &\Leftrightarrow \\
L_j \not\asymp \bigsqcup_{f\in F} \langle f_j\rangle^* L|_{(\operatorname{dom} L)\setminus\{j\}} &\Leftrightarrow \\
L_j \not\asymp \alpha_j L|_{(\operatorname{dom} L)\setminus\{j\}}.&
\end{aligned}$$

□

THEOREM 1683. If f, g are multifuncoids for a primary filtrator $(\mathfrak{A}_i, \mathfrak{Z}_i)$ where $\mathfrak{Z}_i$ are separable starrish posets, then $f \sqcup^{\mathsf{pFCD}(\mathfrak{A})} g \in \mathsf{pFCD}(\mathfrak{A})$.

PROOF. Let $A \in \left[f \sqcup^{\mathsf{pFCD}(\mathfrak{A})} g\right]^*$ and $B \sqsupseteq A$. Then for every $k \in \operatorname{dom}\mathfrak{A}$

$A_k \not\asymp \left\langle f \sqcup^{\mathsf{pFCD}(\mathfrak{A})} g\right\rangle^* A|_{(\operatorname{dom}\mathfrak{A})\setminus\{k\}}$; $A_k \not\asymp \langle f \sqcup g\rangle^* A|_{(\operatorname{dom}\mathfrak{A})\setminus\{k\}}$; $A_k \not\asymp \langle f\rangle^* (A|_{(\operatorname{dom}\mathfrak{A})\setminus\{k\}}) \sqcup \langle g\rangle^* (A|_{(\operatorname{dom}\mathfrak{A})\setminus\{k\}})$.

Thus $A_k \not\asymp \langle f\rangle^*(A|_{(\operatorname{dom}\mathfrak{A})\setminus\{k\}}) \vee A_k \not\asymp \langle g\rangle^*(A|_{(\operatorname{dom}\mathfrak{A})\setminus\{k\}})$; $A \in [f]^* \vee A \in [g]^*$; $B \in [f]^* \vee B \in [g]^*$; $B_k \not\asymp \langle f\rangle^*(B|_{(\operatorname{dom}\mathfrak{A})\setminus\{k\}}) \vee B_k \not\asymp \langle g\rangle^*(B|_{(\operatorname{dom}\mathfrak{A})\setminus\{k\}})$;

$B_k \not\asymp \langle f\rangle^*(B|_{(\operatorname{dom}\mathfrak{A})\setminus\{k\}}) \sqcup \langle g\rangle^*(B|_{(\operatorname{dom}\mathfrak{A})\setminus\{k\}})$; $B_k \not\asymp \langle f \sqcup g\rangle^* B|_{(\operatorname{dom}\mathfrak{A})\setminus\{k\}} = \left\langle f \sqcup^{\mathsf{pFCD}(\mathfrak{A})} g\right\rangle^* B|_{(\operatorname{dom}\mathfrak{A})\setminus\{k\}}$.

Thus $B \in \left[f \sqcup^{\mathsf{pFCD}(\mathfrak{A})} g\right]^*$. □

THEOREM 1684. If F is a set of multifuncoids for the same indexed family of join infinite distributive complete lattices filtrators, then $\bigsqcup^{\mathsf{pFCD}(\mathfrak{A})} F \in \mathsf{pFCD}(\mathfrak{A})$.

PROOF. Let $A \in \left[\bigsqcup^{\mathsf{pFCD}(\mathfrak{A})} F\right]^*$ and $B \sqsupseteq A$. Then for every $k \in \operatorname{dom}\mathfrak{A}$

$A_k \not\asymp \left\langle \bigsqcup^{\mathsf{pFCD}(\mathfrak{A})} F\right\rangle^* A|_{(\operatorname{dom}\mathfrak{A})\setminus\{k\}} = \langle \bigsqcup F\rangle^* A|_{(\operatorname{dom}\mathfrak{A})\setminus\{k\}} = \bigsqcup_{f\in F} \langle f\rangle^*(A|_{(\operatorname{dom}\mathfrak{A})\setminus\{k\}})$.

Thus $\exists f \in F : A_k \not\asymp \langle f\rangle^*(A|_{(\operatorname{dom}\mathfrak{A})\setminus\{k\}})$; $\exists f \in F : A \in [f]^*$; $B \in [f]^*$ for some $f \in F$; $\exists f \in F : B_k \not\asymp \langle f\rangle^*(B|_{(\operatorname{dom}\mathfrak{A})\setminus\{k\}})$; $B_k \not\asymp \bigsqcup_{f\in F} \langle f\rangle^*(B|_{(\operatorname{dom}\mathfrak{A})\setminus\{k\}}) = \left\langle \bigsqcup^{\mathsf{pFCD}(\mathfrak{A})} F\right\rangle^* B|_{(\operatorname{dom}\mathfrak{A})\setminus\{k\}}$. Thus $B \in \left[\bigsqcup^{\mathsf{pFCD}(\mathfrak{A})} F\right]^*$. □