Let now g_0 be a prestaroid, f be a prestaroidal mult corresponding to g_0 by formula (35), and g_1 be a prestaroid corresponding to f by formula (34). Let's prove $g_0 = g_1$. Really,

$$K \in \operatorname{GR} g_1 \Leftrightarrow K_i \in \langle f\rangle_i^* K|_{(\operatorname{dom} L)\setminus\{i\}} \Leftrightarrow K|_{(\operatorname{dom} L)\setminus\{i\}} \cup \{(i, K_i)\} \in \operatorname{GR} g_0 \Leftrightarrow K \in \operatorname{GR} g_0.$$

□

DEFINITION 1671. I will denote $[f]^* = \operatorname{GR} g$ for the prestaroidal mult f corresponding to anchored relation g.

PROPOSITION 1672. For a form $(\mathfrak{Z}, \lambda i \in \operatorname{dom}\mathfrak{Z} : \mathfrak{S}(\mathfrak{Z}_i))$ where each $\mathfrak{Z}_i$ is a boolean lattice, relational mults are the same as multifuncoids (if we equate poset elements with principal free stars).

PROOF.

$$(L_i \not\asymp \langle f\rangle_i^* L|_{(\operatorname{dom} L)\setminus\{i\}} \Leftrightarrow L_j \not\asymp \langle f\rangle_j^* L|_{(\operatorname{dom} L)\setminus\{j\}}) \Leftrightarrow$$
$$(L_i \in \partial\langle f\rangle_i^* L|_{(\operatorname{dom} L)\setminus\{i\}} \Leftrightarrow L_j \in \partial\langle f\rangle_j^* L|_{(\operatorname{dom} L)\setminus\{j\}}) \Leftrightarrow$$
$$(L_i \in \langle f\rangle_i^* L|_{(\operatorname{dom} L)\setminus\{i\}} \Leftrightarrow L_j \in \langle f\rangle_j^* L|_{(\operatorname{dom} L)\setminus\{j\}}).$$

□

THEOREM 1673. Fix some indexed family $\mathfrak{Z}$ of join semi-lattices.

$$(\operatorname{val} f)_j(L \cup \{(i, X \sqcup Y)\}) = (\operatorname{val} f)_j(L \cup \{(i, X)\}) \sqcup (\operatorname{val} f)_j(L \cup \{(i, Y)\})$$

for every prestaroid f of the form $\mathfrak{Z}$ and $i, j \in \operatorname{arity} f$, $i \neq j$, $L \in \prod_{k\in L\setminus\{i,j\}} \mathfrak{Z}_k$, $X, Y \in \mathfrak{Z}_i$.

PROOF. Let $i, j \in \operatorname{arity} f$, $i \neq j$ and $L \in \prod_{k\in L\setminus\{i,j\}} \mathfrak{Z}_k$. Let $Z \in \mathfrak{Z}_i$.

$$\begin{aligned}
Z \in (\operatorname{val} f)_j(L \cup \{(i, X \sqcup Y)\}) &\Leftrightarrow \\
L \cup \{(i, X \sqcup Y), (j, Z)\} \in \operatorname{GR} f &\Leftrightarrow \\
X \sqcup Y \in (\operatorname{val} f)_i(L \cup \{(j, Z)\}) &\Leftrightarrow \\
X \in (\operatorname{val} f)_i(L \cup \{(j, Z)\}) \vee Y \in (\operatorname{val} f)_i(L \cup \{(j, Z)\}) &\Leftrightarrow \\
L \cup \{(i, X), (j, Z)\} \in \operatorname{GR} f \vee L \cup \{(i, Y), (j, Z)\} \in \operatorname{GR} f &\Leftrightarrow \\
Z \in (\operatorname{val} f)_j(L \cup \{(i, X)\}) \vee Z \in (\operatorname{val} f)_j(L \cup \{(i, Y)\}) &\Leftrightarrow \\
Z \in (\operatorname{val} f)_j(L \cup \{(i, X)\}) \cup (\operatorname{val} f)_j(L \cup \{(i, Y)\}) &\Leftrightarrow \\
Z \in (\operatorname{val} f)_j(L \cup \{(i, X)\}) \sqcup (\operatorname{val} f)_j(L \cup \{(i, Y)\}) &
\end{aligned}$$

Thus $(\operatorname{val} f)_j(L \cup \{(i, X \sqcup Y)\}) = (\operatorname{val} f)_j(L \cup \{(i, X)\}) \sqcup (\operatorname{val} f)_j(L \cup \{(i, Y)\})$. □

Let us consider the filtrator $\left(\prod_{i\in\operatorname{arity} f} \mathfrak{S}((\operatorname{form} f)_i), \prod_{i\in\operatorname{arity} f} (\operatorname{form} f)_i\right)$.

CONJECTURE 1674. A finitary anchored relation between join-semilattices is a staroid iff $(\operatorname{val} f)_j(L \cup \{(i, X \sqcup Y)\}) = (\operatorname{val} f)_j(L \cup \{(i, X)\}) \sqcup (\operatorname{val} f)_j(L \cup \{(i, Y)\})$ for every $i, j \in \operatorname{arity} f$ $(i \neq j)$ and $X, Y \in (\operatorname{form} f)_i$.

THEOREM 1675. Let $(\mathfrak{A}_i, \mathfrak{Z}_i)$ be a family of join-closed down-aligned filtrators whose both base and core are join-semilattices. Let f be a staroid of the form $\mathfrak{Z}$. Then $\upuparrows f$ is a staroid of the form $\mathfrak{A}$.

PROOF. First prove that $\upuparrows f$ is a prestaroid. We need to prove that $\bot \notin (\operatorname{GR} \upuparrows f)_i$ (that is $\operatorname{up}\bot \nsubseteq (\operatorname{GR} f)_i$ that is $\bot \notin (\operatorname{GR} f)_i$ what is true by the theorem conditions) and that for every $\mathcal{X}, \mathcal{Y} \in \mathfrak{A}_i$ and $\mathcal{L} \in \prod_{i\in(\operatorname{arity} f)\setminus\{i\}} \mathfrak{A}_i$ where $i \in \operatorname{arity} f$

$$\mathcal{L} \cup \{(i, \mathcal{X} \sqcup \mathcal{Y})\} \in \operatorname{GR} \upuparrows f \Leftrightarrow \mathcal{L} \cup \{(i, \mathcal{X})\} \in \operatorname{GR} \upuparrows f \vee \mathcal{L} \cup \{(i, \mathcal{Y})\} \in \operatorname{GR} \upuparrows f.$$