PROOF.

$$\exists c \in \{0,1\}^n : (\lambda i \in n : L_{c(i)} i) \in \mathrm{GR}\, f \Leftrightarrow$$

$$\exists c \in \{0,1\}^{n-1} : \begin{pmatrix} (\{(n-1, L_0(n-1))\} \cup (\lambda i \in n-1 : L_{c(i)} i)) \in \mathrm{GR}\, f \vee \\ (\{(n-1, L_1(n-1))\} \cup (\lambda i \in n-1 : L_{c(i)} i)) \in \mathrm{GR}\, f \end{pmatrix} \Leftrightarrow$$

$$\exists c \in \{0,1\}^{n-1} : \begin{pmatrix} L_0(n-1) \in (\mathrm{val}\, f)_{n-1}(\lambda i \in n-1 : L_{c(i)} i) \vee \\ L_1(n-1) \in (\mathrm{val}\, f)_{n-1}(\lambda i \in n-1 : L_{c(i)} i) \end{pmatrix} \Leftrightarrow$$

$$\exists c \in \{0,1\}^{n-1} \forall K \in (\mathrm{form}\, f)_i : \begin{pmatrix} K \sqsupseteq L_0(n-1) \vee K \sqsupseteq L_1(n-1) \Rightarrow \\ K \in (\mathrm{val}\, f)_{n-1}(\lambda i \in n-1 : L_{c(i)} i) \end{pmatrix} \Leftrightarrow$$

$$\exists c \in \{0,1\}^{n-1} \forall K \in (\mathrm{form}\, f)_i : \begin{pmatrix} K \sqsupseteq L_0(n-1) \vee K \sqsupseteq L_1(n-1) \Rightarrow \\ \{(n-1, K)\} \cup (\lambda i \in n-1 : L_{c(i)} i) \in \mathrm{GR}\, f \end{pmatrix} \Leftrightarrow$$

$$\ldots$$

$$\forall K \in \prod \mathrm{form}\, f : (K \sqsupseteq L_0 \wedge K \sqsupseteq L_1 \Rightarrow K \in \mathrm{GR}\, f).$$

□

EXERCISE 1644. Prove the simpler special case of the above theorem when the form is a family of join-semilattices.

THEOREM 1645. For finite arity the following are the same:

1°. prestaroids;
2°. staroids;
3°. completary staroids.

PROOF. f is a finitary prestaroid $\Rightarrow$ f is a finitary completary staroid.
f is a finitary completary staroid $\Rightarrow$ f is a finitary staroid.
f is a finitary staroid $\Rightarrow$ f is a finitary prestaroid. □

DEFINITION 1646. We will denote the set of staroids of a form $\mathfrak{A}$ as $\mathsf{Strd}(\mathfrak{A})$.

21.3. Upgrading and downgrading a set regarding a filtrator

Let fix a filtrator $(\mathfrak{A}, \mathfrak{Z})$.

DEFINITION 1647. $\downdownarrows f = f \cap \mathfrak{Z}$ for every $f \in \mathscr{P}\mathfrak{A}$ (*downgrading* f).

DEFINITION 1648. $\upuparrows f = \left\{ \frac{L \in \mathfrak{A}}{\mathrm{up}\, L \subseteq f} \right\}$ for every $f \in \mathscr{P}\mathfrak{Z}$ (*upgrading* f).

OBVIOUS 1649. $a \in \upuparrows f \Leftrightarrow \mathrm{up}\, a \subseteq f$ for every $f \in \mathscr{P}\mathfrak{Z}$ and $a \in \mathfrak{A}$.

PROPOSITION 1650. $\downdownarrows \upuparrows f = f$ if f is an upper set for every $f \in \mathscr{P}\mathfrak{Z}$.

PROOF. $\downdownarrows \upuparrows f = \upuparrows f \cap \mathfrak{Z} = \left\{ \frac{L \in \mathfrak{Z}}{\mathrm{up}\, L \subseteq f} \right\} = \left\{ \frac{L \in \mathfrak{Z}}{L \in f} \right\} = f \cap \mathfrak{Z} = f$. □

21.3.1. Upgrading and downgrading staroids.

Let fix a family $(\mathfrak{A}, \mathfrak{Z})$ of filtrators.

For a graph f of an anchored relation between posets define $\downdownarrows f$ and $\upuparrows f$ taking the filtrator of $(\prod \mathfrak{A}, \prod \mathfrak{Z})$.

For a anchored relation between posets f define:

$$\mathrm{form} \downdownarrows f = \mathfrak{Z} \quad \text{and} \quad \mathrm{GR} \downdownarrows f = \downdownarrows \mathrm{GR}\, f;$$
$$\mathrm{form} \upuparrows f = \mathfrak{A} \quad \text{and} \quad \mathrm{GR} \upuparrows f = \upuparrows \mathrm{GR}\, f.$$

Below we will show that under certain conditions upgraded staroid is a staroid, see theorem 1675.