Definition 1628. *Infinitary anchored relation* is such an anchored relation whose arity is infinite; *finitary anchored relation* is such an anchored relation whose arity is finite.

Definition 1629. An anchored relation *on powersets* is an anchored relation f such that every $(\operatorname{form} f)_i$ is a powerset.

I will denote $\operatorname{arity} f = \operatorname{dom}\operatorname{form} f$.

Definition 1630. $[f]^*$ is the relation between typed elements $\mathfrak{T}(\operatorname{form} f)_i$ (for $i \in \operatorname{arity} f$) defined by the formula $L \in [f]^* \Leftrightarrow \mathfrak{T} \circ L \in \operatorname{GR} f$.

Every set of anchored relations of the same form constitutes a poset by the formula $f \sqsubseteq g \Leftrightarrow \operatorname{GR} f \subseteq \operatorname{GR} g$.

Definition 1631. An anchored relation is an *anchored relation between posets* when every $(\operatorname{form} f)_i$ is a poset.

Definition 1632. $(\operatorname{val} f)_i L = \left\{ \frac{X \in (\operatorname{form} f)_i}{L \cup \{(i,X)\} \in \operatorname{GR} f} \right\}$.

Proposition 1633. f can be restored knowing $\operatorname{form}(f)$ and $(\operatorname{val} f)_i$ for some $i \in \operatorname{arity} f$.

Proof.

$$\operatorname{GR} f = \left\{ \frac{K \in \prod \operatorname{form} f}{K \in \operatorname{GR} f} \right\} = \left\{ \frac{L \cup \{(i,X)\}}{L \in \prod (\operatorname{form} f)|_{(\operatorname{arity} f) \setminus \{i\}}, X \in (\operatorname{form} f)_i, L \cup \{(i,X)\} \in \operatorname{GR} f} \right\} = \left\{ \frac{L \cup \{(i,X)\}}{L \in \prod (\operatorname{form} f)|_{(\operatorname{arity} f) \setminus \{i\}}, X \in (\operatorname{val} f)_i L} \right\}.$$

□

Definition 1634. A *prestaroid* is an anchored relation f between posets such that $(\operatorname{val} f)_i L$ is a free star for every $i \in \operatorname{arity} f$, $L \in \prod (\operatorname{form} f)|_{(\operatorname{arity} f) \setminus \{i\}}$.

Definition 1635. A *staroid* is a prestaroid whose graph is an upper set (on the poset $\prod \operatorname{form}(f)$).

Definition 1636. A *(pre)staroid on power sets* is such a (pre)staroid f that every $(\operatorname{form} f)_i$ is a lattice of all subsets of some set.

Proposition 1637. If $L \in \prod \operatorname{form} f$ and $L_i = \bot^{(\operatorname{form} f)_i}$ for some $i \in \operatorname{arity} f$ then $L \notin \operatorname{GR} f$ if f is a prestaroid.

Proof. Let $K = L|_{(\operatorname{arity} f) \setminus \{i\}}$. We have $\bot \notin (\operatorname{val} f)_i K$; $K \cup \{(i, \bot)\} \notin \operatorname{GR} f$; $L \notin \operatorname{GR} f$. □

Next we will define *completary staroids*. First goes the general case, next simpler case for the special case of join-semilattices instead of arbitrary posets.

Definition 1638. A *completary staroid* is an anchored relation between posets conforming to the formulas:

1°. $\forall K \in \prod \operatorname{form} f : (K \sqsupseteq L_0 \wedge K \sqsupseteq L_1 \Rightarrow K \in \operatorname{GR} f)$ is equivalent to $\exists c \in \{0,1\}^n : (\lambda i \in n : L_{c(i)} i) \in \operatorname{GR} f$ for every $L_0, L_1 \in \prod \operatorname{form} f$.

2°. If $L \in \prod \operatorname{form} f$ and $L_i = \bot^{(\operatorname{form} f)_i}$ for some $i \in \operatorname{arity} f$ then $L \notin \operatorname{GR} f$.

Lemma 1639. Every graph of completary staroid is an upper set.

Proof. Let f be a completary staroid. Let $L_0 \sqsubseteq L_1$ for some $L_0, L_1 \in \prod \operatorname{form} f$ and $L_0 \in \operatorname{GR} f$. Then taking $c = n \times \{0\}$ we get $\lambda i \in n : L_{c(i)} i = \lambda i \in n : L_0 i = L_0 \in \operatorname{GR} f$ and thus $L_1 \in \operatorname{GR} f$ because $L_1 \sqsupseteq L_0 \wedge L_1 \sqsupseteq L_1$. □