THEOREM 1625. Let f, g be pointfree funcoids between filters on boolean lattices. Then for every filters $\mathcal{A}_0 \in \mathscr{F}(\operatorname{Src} f)$, $\mathcal{B}_0 \in \mathscr{F}(\operatorname{Src} g)$

$$\left\langle f \times^{(C)} g \right\rangle (\mathcal{A}_0 \times^{\mathsf{FCD}} \mathcal{B}_0) = \langle f \rangle \mathcal{A}_0 \times^{\mathsf{FCD}} \langle g \rangle \mathcal{B}_0.$$

PROOF. For every atom $a_1 \times^{\mathsf{FCD}} b_1$ ($a_1 \in \operatorname{atoms}^{\operatorname{Dst} f}$, $b_1 \in \operatorname{atoms}^{\operatorname{Dst} g}$) (see theorem 1569) of the lattice of funcoids we have:

$$\begin{aligned}
a_1 \times^{\mathsf{FCD}} b_1 \not\asymp \left\langle f \times^{(C)} g \right\rangle (\mathcal{A}_0 \times^{\mathsf{FCD}} \mathcal{B}_0) &\Leftrightarrow \\
\mathcal{A}_0 \times^{\mathsf{FCD}} \mathcal{B}_0 \left[f \times^{(C)} g \right] a_1 \times^{\mathsf{FCD}} b_1 &\Leftrightarrow \\
(\mathcal{A}_0 \times^{\mathsf{FCD}} \mathcal{B}_0) \circ f^{-1} \not\asymp g^{-1} \circ (a_1 \times^{\mathsf{FCD}} b_1) &\Leftrightarrow \\
\langle f \rangle \mathcal{A}_0 \times^{\mathsf{FCD}} \mathcal{B}_0 \not\asymp a_1 \times^{\mathsf{FCD}} \langle g^{-1} \rangle b_1 &\Leftrightarrow \\
\langle f \rangle \mathcal{A}_0 \not\asymp a_1 \wedge \langle g^{-1} \rangle b_1 \not\asymp \mathcal{B}_0 &\Leftrightarrow \\
\langle f \rangle \mathcal{A}_0 \not\asymp a_1 \wedge \langle g \rangle \mathcal{B}_0 \not\asymp b_1 &\Leftrightarrow \\
\langle f \rangle \mathcal{A}_0 \times^{\mathsf{FCD}} \langle g \rangle \mathcal{B}_0 \not\asymp a_1 \times^{\mathsf{FCD}} b_1&.
\end{aligned}$$

Thus $\left\langle f \times^{(C)} g \right\rangle (\mathcal{A}_0 \times^{\mathsf{FCD}} \mathcal{B}_0) = \langle f \rangle \mathcal{A}_0 \times^{\mathsf{FCD}} \langle g \rangle \mathcal{B}_0$ because the lattice $\mathsf{pFCD}(\mathscr{F}(\operatorname{Dst} f), \mathscr{F}(\operatorname{Dst} g))$ is atomically separable (corollary 1560). □

COROLLARY 1626. $\mathcal{A}_0 \times^{\mathsf{FCD}} \mathcal{B}_0 \left[f \times^{(C)} g \right] \mathcal{A}_1 \times^{\mathsf{FCD}} \mathcal{B}_1 \Leftrightarrow \mathcal{A}_0 [f] \mathcal{A}_1 \wedge \mathcal{B}_0 [g] \mathcal{B}_1$ for every $\mathcal{A}_0 \in \mathscr{F}(\operatorname{Src} f)$, $\mathcal{A}_1 \in \mathscr{F}(\operatorname{Dst} f)$, $\mathcal{B}_0 \in \mathscr{F}(\operatorname{Src} g)$, $\mathcal{B}_1 \in \mathscr{F}(\operatorname{Dst} g)$ where $\operatorname{Src} f$, $\operatorname{Dst} f$, $\operatorname{Src} g$, $\operatorname{Dst} g$ are boolean lattices.

PROOF.

$$\begin{aligned}
\mathcal{A}_0 \times^{\mathsf{FCD}} \mathcal{B}_0 \left[f \times^{(C)} g \right] \mathcal{A}_1 \times^{\mathsf{FCD}} \mathcal{B}_1 &\Leftrightarrow \\
\mathcal{A}_1 \times^{\mathsf{FCD}} \mathcal{B}_1 \not\asymp \left\langle f \times^{(C)} g \right\rangle \mathcal{A}_0 \times^{\mathsf{FCD}} \mathcal{B}_0 &\Leftrightarrow \\
\mathcal{A}_1 \times^{\mathsf{FCD}} \mathcal{B}_1 \not\asymp \langle f \rangle \mathcal{A}_0 \times^{\mathsf{FCD}} \langle g \rangle \mathcal{B}_0 &\Leftrightarrow \\
\mathcal{A}_1 \not\asymp \langle f \rangle \mathcal{A}_0 \wedge \mathcal{B}_1 \not\asymp \langle g \rangle \mathcal{B}_0 &\Leftrightarrow \\
\mathcal{A}_0 [f] \mathcal{A}_1 \wedge \mathcal{B}_0 [g] \mathcal{B}_1&.
\end{aligned}$$

□

21.2. Definition of staroids

It follows from the above theorem 828 that funcoids are essentially the same as relations δ between sets A and B, such that $\left\{ \frac{Y \in \mathscr{P} B}{\exists X \in \mathscr{P} A : X \delta Y} \right\}$ and $\left\{ \frac{X \in \mathscr{P} A}{\exists Y \in \mathscr{P} B : X \delta Y} \right\}$ are free stars. This inspires the below definition of staroids (switching from two sets X and Y to a (potentially infinite) family of posets).

Whilst I have (mostly) thoroughly studied basic properties of funcoids, *staroids* (defined below) are yet much a mystery. For example, we do not know whether the set of staroids on powersets is atomic.

Let n be a set. As an example, n may be an ordinal, n may be a natural number, considered as a set by the formula $n = \{0, \ldots, n-1\}$. Let $\mathfrak{A} = \mathfrak{A}_{i \in n}$ be a family of posets indexed by the set n.

DEFINITION 1627. I will call an *anchored relation* a pair $f = (\operatorname{form} f, \operatorname{GR} f)$ of a family $\operatorname{form}(f)$ of relational structures indexed by some index set and a relation $\operatorname{GR}(f) \in \mathscr{P} \prod \operatorname{form}(f)$. I call $\operatorname{GR}(f)$ the *graph* of the anchored relation f. I denote $\operatorname{Anch}(\mathfrak{A})$ the set of anchored relations of the form $\mathfrak{A}$.