That Φ_2^{-1} preserves composition and identities follows from the fact that it is an isomorphism.

That is preserves reversal follows from the formula $\langle f^{-1}\rangle = \langle p^{-1}\rangle^*$. □

PROPOSITION 1614. The bijections Ψ_2 and Ψ_2^{-1} from the diagram at figure 1 preserves monovaluedness and injectivity.

PROOF. Because it is a functor which preserves reversal. □

PROPOSITION 1615. The bijections Ψ_2 and Ψ_2^{-1} from the diagram at figure 1 preserves domain an image.

PROOF. $\operatorname{im} f = \langle f\rangle\top = \langle p\rangle^*\top = \operatorname{im} p$, likewise for domain. □

PROPOSITION 1616. The bijections Ψ_2 and Ψ_2^{-1} from the diagram at figure 1 maps cartesian products to corresponding funcoidal products.

PROOF. $\langle A \times^{\mathsf{FCD}} B\rangle X = \begin{cases} B & \text{if } X \not\asymp A \\ \bot & \text{if } X \asymp A \end{cases} = \langle A \times B\rangle^* X$. Likewise $\langle (A \times^{\mathsf{FCD}} B)^{-1}\rangle Y = \langle (A \times B)^{-1}\rangle^* Y$. □

Let Φ map a pointfree funcoid whose first component is c into the Galois connection whose lower adjoint is c. Then Φ is an isomorphism (theorem 1604) and Φ^{-1} maps a Galois connection whose lower adjoint is c into the pointfree funcoid whose first component is c.

Informally speaking, Φ replaces a relation r with its complement relations $\neg r$. Formally:

PROPOSITION 1617.

1°. For every path P in the diagram at figure 1 from binary relations between A and B to pointfree funcoids between $\mathscr{P}A$ and $\mathscr{P}B$ and every path Q in the diagram at figure 1 from Galois connections between $\mathscr{P}A$ and $\mathscr{P}B$ to binary relations between A and B, we have $Q\Phi Pr = \neg r$.

2°. For every path Q in the diagram at figure 1 from binary relations between A and B to pointfree funcoids between $\mathscr{P}A$ and $\mathscr{P}B$ and every path P in the diagram at figure 1 from Galois connections between $\mathscr{P}A$ and $\mathscr{P}B$ to binary relations between A and B, we have $P\Phi^{-1}Qr = \neg r$.

PROOF. We will prove only the second ($P \circ \Phi^{-1} \circ Q = \neg$), because the first ($Q \circ \Phi \circ P = \neg$) can be obtained from it by inverting the morphisms (and variable replacement).

Because the diagram is commutative, it is enough to prove it for some fixed P and Q. For example, we will prove $\Psi_2^{-1}\Phi^{-1}\Psi_4\Psi_2 r = \neg r$.

$\Psi_4\Psi_2 r = \left(X \mapsto \neg\bigsqcap_{x\in X}\langle r\rangle^*\{x\}, Y \mapsto \bigsqcap_{y\in Y}\langle r\rangle^*\neg\{y\}\right)$.

$\Phi^{-1}\Psi_4\Psi_2 r$ is pointfree funcoid f with $\langle f\rangle = X \mapsto \neg\bigsqcap_{x\in X}\langle r\rangle^*\{x\}$.

$\Psi_2^{-1}\Phi^{-1}\Psi_4\Psi_2 r$ is the relation consisting of (x, y) such that $\{x\}\ [f]\ \{y\}$ what is equivalent to: $\{y\} \not\asymp \langle f\rangle\{x\}$; $\{y\} \not\asymp \neg\langle r\rangle^*\{x\}$; $\{y\} \not\sqsubseteq \langle r\rangle^*\{x\}$; $y \notin \langle r\rangle^*\{x\}$.

So $\Psi_2^{-1}\Phi^{-1}\Psi_4\Psi_2 r = \neg r$. □

PROPOSITION 1618. Φ and Φ^{-1} preserve composition.

PROOF. By definitions of compositions and the fact that both pointfree funcoids and Galois connections are determined by the first component. □