



$$\begin{aligned} \Psi_1. f &\mapsto \left\{ \frac{(x,y)}{y \in f_0\{x\}} \right\} = \left\{ \frac{(x,y)}{x \in f_1\{y\}} \right\} \\ \Psi_1^{-1}. r &\mapsto \left(X \mapsto \left\{ \frac{y \in B}{\forall x \in X: x r y} \right\}, Y \mapsto \left\{ \frac{x \in A}{\forall y \in Y: x r y} \right\} \right) \\ \Psi_2. r &\mapsto (\mathcal{P}A, \mathcal{P}B, \langle r \rangle^*, \langle r^{-1} \rangle^*) \\ \Psi_2^{-1}. f &\mapsto \left\{ \frac{(x,y)}{\{x\}[f]\{y\}} \right\} \\ \Psi_3. f &\mapsto \left(X \mapsto \prod_{x \in \mathcal{T}X \setminus \{\perp\}} \langle f \rangle x, Y \mapsto \prod_{y \in \mathcal{T}Y \setminus \{\perp\}} \langle f^{-1} \rangle y \right) = \\ &\quad \left(X \mapsto \prod_{x \in X} \langle f \rangle \{x\}, Y \mapsto \prod_{y \in Y} \langle f^{-1} \rangle \{y\} \right) \\ \Psi_3^{-1}. f &\mapsto \left(\mathcal{P}A, \mathcal{P}B, X \mapsto \bigsqcup_{x \in \mathcal{T}X \setminus \{\perp\}} f_0 x, Y \mapsto \bigsqcup_{y \in \mathcal{T}Y \setminus \{\perp\}} f_1 y \right) = \\ &\quad \left(\mathcal{P}A, \mathcal{P}B, X \mapsto \bigsqcup_{x \in X} f_0 \{x\}, Y \mapsto \bigsqcup_{y \in Y} f_1 \{y\} \right) \\ \Psi_4. f &\mapsto \left(X \mapsto \neg \prod_{x \in \mathcal{T}X \setminus \{\perp\}} \langle f \rangle x, Y \mapsto \prod_{y \in \mathcal{T}Y \setminus \{\perp\}} \langle f^{-1} \rangle \neg y \right) = \\ &\quad \left(X \mapsto \bigsqcup_{x \in \mathcal{T}X \setminus \{\perp\}} \neg \langle f \rangle x, Y \mapsto \prod_{y \in \mathcal{T}Y \setminus \{\perp\}} \langle f^{-1} \rangle \neg y \right) = \\ &\quad \left(X \mapsto \neg \prod_{x \in X} \langle f \rangle \{x\}, Y \mapsto \prod_{y \in Y} \langle f^{-1} \rangle \neg \{y\} \right) = \\ &\quad \left(X \mapsto \bigsqcup_{x \in X} \neg \langle f \rangle \{x\}, Y \mapsto \prod_{y \in Y} \langle f^{-1} \rangle \neg \{y\} \right) \\ \Psi_4^{-1}. f &\mapsto \left(\mathcal{P}A, \mathcal{P}B, X \mapsto \bigsqcup_{x \in \mathcal{T}X \setminus \{\perp\}} \neg f_0 x, Y \mapsto \bigsqcup_{y \in \mathcal{T}Y \setminus \{\perp\}} f_1 \neg y \right) = \\ &\quad \left(\mathcal{P}A, \mathcal{P}B, X \mapsto \neg \prod_{x \in \mathcal{T}X \setminus \{\perp\}} f_0 x, Y \mapsto \bigsqcup_{y \in \mathcal{T}Y \setminus \{\perp\}} f_1 \neg y \right) = \\ &\quad \left(\mathcal{P}A, \mathcal{P}B, X \mapsto \bigsqcup_{x \in X} \neg f_0 \{x\}, Y \mapsto \bigsqcup_{y \in Y} f_1 \neg \{y\} \right) = \\ &\quad \left(\mathcal{P}A, \mathcal{P}B, X \mapsto \neg \prod_{x \in X} f_0 \{x\}, Y \mapsto \bigsqcup_{y \in Y} f_1 \neg \{y\} \right) \\ \Psi_5 = \Psi_5^{-1}. f &\mapsto (\neg \circ f_0, f_1 \circ \neg) \end{aligned}$$

Claim: Ψ_1 maps antitone Galois connections between $\mathcal{P}A$ and $\mathcal{P}B$ into binary relations between A and B .

Proof: Obvious. ■

Claim: Ψ_1^{-1} maps binary relations between A and B into antitone Galois connections between $\mathcal{P}A$ and $\mathcal{P}B$.

Proof: We need to prove $Y \subseteq \left\{ \frac{y \in B}{\forall x \in X: x r y} \right\} \Leftrightarrow X \subseteq \left\{ \frac{x \in A}{\forall y \in Y: x r y} \right\}$. After we equivalently rewrite it:

$$\forall y \in Y \forall x \in X : x r y \Leftrightarrow \forall x \in X \forall y \in Y : x r y$$