b); $b \sqcap \langle f \rangle \langle f^{-1} \rangle a = \bot^{\mathfrak{B}}$; $b \sqcap \langle f \circ f^{-1} \rangle a = \bot^{\mathfrak{B}}$; $\neg (a \left[f \circ f^{-1} \right] b)$. So $a \left[f \circ f^{-1} \right] b \Rightarrow a = b$ for every $a, b \in \operatorname{atoms}^{\mathfrak{B}}$. This is possible only (corollary 1544 and the fact that $\mathfrak{B}$ is atomic) when $f \circ f^{-1} \sqsubseteq 1^{\mathsf{pFCD}}_{\mathfrak{B}}$.

$\neg 2° \Rightarrow \neg 1°$. Suppose $\langle f \rangle a \notin \operatorname{atoms}^{\mathfrak{B}} \cup \{\bot^{\mathfrak{B}}\}$ for some $a \in \operatorname{atoms}^{\mathfrak{A}}$. Then there exist two atoms $p \neq q$ such that $\langle f \rangle a \sqsupseteq p \wedge \langle f \rangle a \sqsupseteq q$. Consequently $p \sqcap \langle f \rangle a \neq \bot^{\mathfrak{B}}$; $a \sqcap \langle f^{-1} \rangle p \neq \bot^{\mathfrak{A}}$; $a \sqsubseteq \langle f^{-1} \rangle p$; $\langle f \circ f^{-1} \rangle p = \langle f \rangle \langle f^{-1} \rangle p \sqsupseteq \langle f \rangle a \sqsupseteq q$ (by proposition 1497 because $\mathfrak{B}$ is separable by proposition 231 and thus strongly separable by theorem 222); $\langle f \circ f^{-1} \rangle p \not\sqsubseteq p$ and $\langle f \circ f^{-1} \rangle p \neq \bot^{\mathfrak{B}}$. So it cannot be $f \circ f^{-1} \sqsubseteq 1^{\mathsf{pFCD}}_{\mathfrak{B}}$.

□

THEOREM 1591. The following is equivalent for primary filtrators $(\mathfrak{A}, \mathfrak{Z}_0)$ and $(\mathfrak{B}, \mathfrak{Z}_1)$ over boolean lattices and pointfree funcoids $f : \mathfrak{A} \to \mathfrak{B}$:

1°. f is monovalued.
2°. f is metamonovalued.
3°. f is weakly metamonovalued.

PROOF.

2°⇒3°. Obvious.
1°⇒2°.

$$\left\langle \left(\bigsqcap G \right) \circ f \right\rangle x = \left\langle \left(\bigsqcap G \right) \right\rangle \langle f \rangle x = \bigsqcap_{g \in G} \langle g \rangle \langle f \rangle x = \bigsqcap_{g \in G} \langle g \circ f \rangle x = \left\langle \bigsqcap_{g \in G} (g \circ f) \right\rangle x$$

for every atomic filter object $x \in \operatorname{atoms}^{\mathfrak{A}}$. Thus $(\bigsqcap G) \circ f = \bigsqcap_{g \in G} (g \circ f)$.

3°⇒1°. Take $g = a \times^{\mathsf{FCD}} y$ and $h = b \times^{\mathsf{FCD}} y$ for arbitrary atomic filter objects $a \neq b$ and y. We have $g \sqcap h = \bot$; thus $(g \circ f) \sqcap (h \circ f) = (g \sqcap h) \circ f = \bot$ and thus impossible $x \, [f] \, a \wedge x \, [f] \, b$ as otherwise $x \, [g \circ f] \, y$ and $x \, [h \circ f] \, y$ so $x \, [(g \circ f) \sqcap (h \circ f)] \, y$. Thus f is monovalued.

□

THEOREM 1592. Let $(\mathfrak{A}, \mathfrak{Z}_0)$ and $(\mathfrak{B}, \mathfrak{Z}_1)$ be primary filtrators over boolean lattices. A pointfree funcoid $f \in \mathsf{pFCD}(\mathfrak{A}, \mathfrak{B})$ is monovalued iff

$$\forall I, J \in \mathfrak{Z}_1 : \langle f^{-1} \rangle (I \sqcap^{\mathfrak{Z}_1} J) = \langle f^{-1} \rangle I \sqcap \langle f^{-1} \rangle J.$$

PROOF. $\mathfrak{A}$ and $\mathfrak{B}$ are complete lattices (corollary 515).
$(\mathfrak{B}, \mathfrak{Z}_1)$ is a filtrator with separable core by theorem 534.
$(\mathfrak{B}, \mathfrak{Z}_1)$ is binarily meet-closed by corollary 533.
$\mathfrak{A}$ and $\mathfrak{B}$ are starrish by corollary 528.
$(\mathfrak{A}, \mathfrak{Z}_0)$ is with separable core by theorem 534.
We are under conditions of theorem 1509 for the pointfree funcoid f^{-1}.

⇒. Obvious (taking into account that $(\mathfrak{B}, \mathfrak{Z}_1)$ is binarily meet-closed).