If $x \sqcap \bigsqcap \operatorname{dom} S \neq \bot^{\mathfrak{A}}$ then

$$\forall (\mathcal{A}, \mathcal{B}) \in S : (x \sqcap \mathcal{A} \neq \bot^{\mathfrak{A}} \wedge \langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \rangle x = \mathcal{B});$$

$$\left\{ \frac{\langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \rangle x}{(\mathcal{A}, \mathcal{B}) \in S} \right\} = \operatorname{im} S;$$

if $x \sqcap \bigsqcap \operatorname{dom} S = \bot^{\mathfrak{A}}$ then

$$\exists (\mathcal{A}, \mathcal{B}) \in S : (x \sqcap \mathcal{A} = \bot^{\mathfrak{A}} \wedge \langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \rangle x = \bot^{\mathcal{B}});$$

$$\left\{ \frac{\langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \rangle x}{(\mathcal{A}, \mathcal{B}) \in S} \right\} \ni \bot^{\mathfrak{B}}.$$

So

$$\left\langle \bigsqcap_{(\mathcal{A}, \mathcal{B}) \in S} (\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}) \right\rangle x = \begin{cases} \bigsqcap \operatorname{im} S & \text{if } x \sqcap \bigsqcap \operatorname{dom} S \neq \bot^{\mathfrak{A}}; \\ \bot^{\mathfrak{B}} & \text{if } x \sqcap \bigsqcap \operatorname{dom} S = \bot^{\mathfrak{A}}. \end{cases}$$

From this by theorem 1546 the statement of our theorem follows. □

COROLLARY 1564. *Let $\mathfrak{A}$ and $\mathfrak{B}$ be posets of filters over boolean lattices. For every $\mathcal{A}_0, \mathcal{A}_1 \in \mathfrak{A}$ and $\mathcal{B}_0, \mathcal{B}_1 \in \mathfrak{B}$*

$$(\mathcal{A}_0 \times^{\mathsf{FCD}} \mathcal{B}_0) \sqcap (\mathcal{A}_1 \times^{\mathsf{FCD}} \mathcal{B}_1) = (\mathcal{A}_0 \sqcap \mathcal{A}_1) \times^{\mathsf{FCD}} (\mathcal{B}_0 \sqcap \mathcal{B}_1).$$

PROOF. $(\mathcal{A}_0 \times^{\mathsf{FCD}} \mathcal{B}_0) \sqcap (\mathcal{A}_1 \times^{\mathsf{FCD}} \mathcal{B}_1) = \bigsqcap \{\mathcal{A}_0 \times^{\mathsf{FCD}} \mathcal{B}_0, \mathcal{A}_1 \times^{\mathsf{FCD}} \mathcal{B}_1\}$ what is by the last theorem equal to $(\mathcal{A}_0 \sqcap \mathcal{A}_1) \times^{\mathsf{FCD}} (\mathcal{B}_0 \sqcap \mathcal{B}_1)$. □

THEOREM 1565. *Let $(\mathfrak{A}, \mathfrak{Z}_0)$ and $(\mathfrak{B}, \mathfrak{Z}_1)$ be primary filtrators over boolean lattices. If $\mathcal{A} \in \mathfrak{A}$ then $\mathcal{A} \times^{\mathsf{FCD}}$ is a complete homomorphism from the lattice $\mathfrak{A}$ to the lattice $\mathsf{pFCD}(\mathfrak{A}, \mathfrak{B})$, if also $\mathcal{A} \neq \bot^{\mathfrak{A}}$ then it is an order embedding.*

PROOF. Let $S \in \mathscr{P}\mathfrak{A}$, $X \in \mathfrak{Z}_0$, $x \in \operatorname{atoms}^{\mathfrak{A}}$.

$$\left\langle \bigsqcup \langle \mathcal{A} \times^{\mathsf{FCD}} \rangle^* S \right\rangle X =$$

$$\bigsqcup_{\mathcal{B} \in S} \langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \rangle X =$$

$$\begin{cases} \bigsqcup S & \text{if } X \sqcap^{\mathfrak{A}} \mathcal{A} \neq \bot^{\mathfrak{A}} \\ \bot^{\mathfrak{B}} & \text{if } X \sqcap^{\mathfrak{A}} \mathcal{A} = \bot^{\mathfrak{A}} \end{cases} =$$

$$\left\langle \mathcal{A} \times^{\mathsf{FCD}} \bigsqcup S \right\rangle X.$$

Thus $\bigsqcup \langle \mathcal{A} \times^{\mathsf{FCD}} \rangle^* S = \mathcal{A} \times^{\mathsf{FCD}} \bigsqcup S$ by theorem 1509.

$$\left\langle \bigsqcap \langle \mathcal{A} \times^{\mathsf{FCD}} \rangle^* S \right\rangle x =$$

$$\bigsqcap_{\mathcal{B} \in S} \langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \rangle x =$$

$$\begin{cases} \bigsqcap S & \text{if } X \sqcap^{\mathfrak{A}} \mathcal{A} \neq \bot^{\mathfrak{A}} \\ \bot^{\mathfrak{B}} & \text{if } X \sqcap^{\mathfrak{A}} \mathcal{A} = \bot^{\mathfrak{A}} \end{cases} =$$

$$\left\langle \mathcal{A} \times^{\mathsf{FCD}} \bigsqcap S \right\rangle x.$$

Thus $\bigsqcap \langle \mathcal{A} \times^{\mathsf{FCD}} \rangle^* S = \mathcal{A} \times^{\mathsf{FCD}} \bigsqcap S$ by theorem 1542.

If $\mathcal{A} \neq \bot^{\mathfrak{A}}$ then obviously $\mathcal{A} \times^{\mathsf{FCD}} \mathcal{X} \sqsubseteq \mathcal{A} \times^{\mathsf{FCD}} \mathcal{Y} \Leftrightarrow \mathcal{X} \sqsubseteq \mathcal{Y}$, because $\operatorname{im}(\mathcal{A} \times^{\mathsf{FCD}} \mathcal{X}) = \mathcal{X}$ and $\operatorname{im}(\mathcal{A} \times^{\mathsf{FCD}} \mathcal{Y}) = \mathcal{Y}$. □