PROOF. Existence of no more than one such funcoids and formulas (25) and (27) follow from theorem 1542 and corollary 1544 and the fact that our filtrators are separable.

1°. Consider the function $\alpha' \in \mathfrak{B}^{\mathfrak{Z}_0}$ defined by the formula (for every $X \in \mathfrak{Z}_0$)

$$\alpha' X = \bigsqcup \langle \alpha \rangle^* \operatorname{atoms}^{\mathfrak{A}} X.$$

Obviously $\alpha' \bot^{\mathfrak{Z}_0} = \bot^{\mathfrak{B}}$. For every $I, J \in \mathfrak{Z}_0$

$$\begin{aligned} \alpha'(I \sqcup J) &= \\ \bigsqcup \langle \alpha \rangle^* \operatorname{atoms}^{\mathfrak{A}}(I \sqcup J) &= \\ \bigsqcup \langle \alpha \rangle^* (\operatorname{atoms}^{\mathfrak{A}} I \cup \operatorname{atoms}^{\mathfrak{A}} J) &= \\ \bigsqcup (\langle \alpha \rangle^* \operatorname{atoms}^{\mathfrak{A}} I \cup \langle \alpha \rangle^* \operatorname{atoms}^{\mathfrak{A}} J) &= \\ \bigsqcup \langle \alpha \rangle^* \operatorname{atoms}^{\mathfrak{A}} I \sqcup \bigsqcup \langle \alpha \rangle^* \operatorname{atoms}^{\mathfrak{A}} J &= \\ \alpha' I \sqcup \alpha' J. \end{aligned}$$

Let continue α' till a pointfree funcoid f (by the theorem 1510): $\langle f \rangle \mathcal{X} = \bigsqcap \langle \alpha' \rangle^* \operatorname{up}^{\mathfrak{Z}_0} \mathcal{X}$.

Let's prove the reverse of (24):

$$\begin{aligned} \bigsqcap \left\langle \bigsqcup \circ \langle \alpha \rangle^* \circ \operatorname{atoms}^{\mathfrak{A}} \right\rangle^* \operatorname{up}^{\mathfrak{Z}_0} a &= \\ \bigsqcap \left\langle \bigsqcup \circ \langle \alpha \rangle^* \right\rangle^* \left\langle \operatorname{atoms}^{\mathfrak{A}} \right\rangle^* \operatorname{up}^{\mathfrak{Z}_0} a &\sqsubseteq \\ \bigsqcap \left\langle \bigsqcup \circ \langle \alpha \rangle^* \right\rangle^* \{\{a\}\} &= \\ \bigsqcap \left\{ \left(\bigsqcup \circ \langle \alpha \rangle^* \right) \{a\} \right\} &= \\ \bigsqcap \left\{ \bigsqcup \langle \alpha \rangle^* \{a\} \right\} &= \\ \bigsqcap \left\{ \bigsqcup \{\alpha a\} \right\} &= \\ \bigsqcap \{\alpha a\} = \alpha a. \end{aligned}$$

Finally,

$$\alpha a = \bigsqcap \left\langle \bigsqcup \circ \langle \alpha \rangle^* \circ \operatorname{atoms}^{\mathfrak{A}} \right\rangle^* \operatorname{up}^{\mathfrak{Z}_0} a = \bigsqcap \langle \alpha' \rangle^* \operatorname{up}^{\mathfrak{Z}_0} a = \langle f \rangle a,$$

so $\langle f \rangle$ is a continuation of α.

2°. Consider the relation $\delta' \in \mathscr{P}(\mathfrak{Z}_0 \times \mathfrak{Z}_1)$ defined by the formula (for every $X \in \mathfrak{Z}_0$, $Y \in \mathfrak{Z}_1$)

$$X \mathrel{\delta'} Y \Leftrightarrow \exists x \in \operatorname{atoms}^{\mathfrak{A}} X, y \in \operatorname{atoms}^{\mathfrak{B}} Y : x \mathrel{\delta} y.$$

Obviously $\neg(X \mathrel{\delta'} \bot^{\mathfrak{Z}_1})$ and $\neg(\bot^{\mathfrak{Z}_0} \mathrel{\delta'} Y)$.

$$\begin{aligned} I \sqcup J \mathrel{\delta'} Y &\Leftrightarrow \\ \exists x \in \operatorname{atoms}^{\mathfrak{A}}(I \sqcup J), y \in \operatorname{atoms}^{\mathfrak{B}} Y : x \mathrel{\delta} y &\Leftrightarrow \\ \exists x \in \operatorname{atoms}^{\mathfrak{A}} I \cup \operatorname{atoms}^{\mathfrak{A}} J, y \in \operatorname{atoms}^{\mathfrak{B}} Y : x \mathrel{\delta} y &\Leftrightarrow \\ \exists x \in \operatorname{atoms}^{\mathfrak{A}} I, y \in \operatorname{atoms}^{\mathfrak{B}} Y : x \mathrel{\delta} y \vee \exists x \in \operatorname{atoms}^{\mathfrak{A}} J, y \in \operatorname{atoms}^{\mathfrak{B}} Y : x \mathrel{\delta} y &\Leftrightarrow \\ I \mathrel{\delta'} Y \vee J \mathrel{\delta'} Y; \end{aligned}$$