PROPOSITION 1537. $\langle f\rangle x = \langle f\rangle(x \sqcap \operatorname{dom} f)$ for every $x \in \operatorname{Src} f$ for a pointfree funcoid f whose source is a bounded separable meet-semilattice and destination is a bounded separable poset.

PROOF. $\operatorname{Src} f$ is strongly separable by theorem 222. For every $y \in \operatorname{Dst} f$ we have

$$y \not\asymp \langle f\rangle(x \sqcap \operatorname{dom} f) \Leftrightarrow x \sqcap \operatorname{dom} f \sqcap \langle f^{-1}\rangle y \neq \bot^{\operatorname{Src} f} \Leftrightarrow$$
$$x \sqcap \operatorname{im} f^{-1} \sqcap \langle f^{-1}\rangle y \neq \bot^{\operatorname{Src} f} \Leftrightarrow$$
$$\text{(by strong separability of } \operatorname{Src} f)$$
$$x \sqcap \langle f^{-1}\rangle y \neq \bot^{\operatorname{Src} f} \Leftrightarrow y \not\asymp \langle f\rangle x.$$

Thus $\langle f\rangle x = \langle f\rangle(x \sqcap \operatorname{dom} f)$ by separability of $\operatorname{Dst} f$. □

PROPOSITION 1538. $x \not\asymp \operatorname{dom} f \Leftrightarrow (\langle f\rangle x$ is not least) for every pointfree funcoid f and $x \in \operatorname{Src} f$ if $\operatorname{Dst} f$ has greatest element $\top$.

PROOF. $x \not\asymp \operatorname{dom} f \Leftrightarrow x \not\asymp \langle f^{-1}\rangle \top^{\operatorname{Dst} f} \Leftrightarrow \top^{\operatorname{Dst} f} \not\asymp \langle f\rangle x \Leftrightarrow$ $(\langle f\rangle x$ is not least). □

PROPOSITION 1539. $\operatorname{dom} f = \bigsqcup\left\{\frac{a \in \operatorname{atoms}^{\operatorname{Src} f}}{\langle f\rangle a \neq \bot^{\operatorname{Dst} f}}\right\}$ for every pointfree funcoid f whose destination is a bounded strongly separable poset and source is an atomistic poset.

PROOF. For every $a \in \operatorname{atoms}^{\operatorname{Src} f}$ we have

$$a \not\asymp \operatorname{dom} f \Leftrightarrow a \not\asymp \langle f^{-1}\rangle \top^{\operatorname{Dst} f} \Leftrightarrow \top^{\operatorname{Dst} f} \not\asymp \langle f\rangle a \Leftrightarrow \langle f\rangle a \neq \bot^{\operatorname{Dst} f}.$$

So $\operatorname{dom} f = \bigsqcup\left\{\frac{a \in \operatorname{atoms}^{\operatorname{Src} f}}{a \not\asymp \operatorname{dom} f}\right\} = \bigsqcup\left\{\frac{a \in \operatorname{atoms}^{\operatorname{Src} f}}{\langle f\rangle a \neq \bot^{\operatorname{Dst} f}}\right\}$. □

PROPOSITION 1540. $\operatorname{dom}(f|_a) = a \sqcap \operatorname{dom} f$ for every pointfree funcoid f and $a \in \operatorname{Src} f$ where $\operatorname{Src} f$ is a meet-semilattice and $\operatorname{Dst} f$ has greatest element.

PROOF.

$$\operatorname{dom}(f|_a) = \operatorname{im}(\mathrm{id}_a^{\mathsf{pFCD}(\operatorname{Src} f)} \circ f^{-1}) =$$
$$\left\langle \mathrm{id}_a^{\mathsf{pFCD}(\operatorname{Src} f)} \right\rangle \langle f^{-1}\rangle \top^{\operatorname{Dst} f} = a \sqcap \langle f^{-1}\rangle \top^{\operatorname{Dst} f} = a \sqcap \operatorname{dom} f.$$

□

PROPOSITION 1541. For every composable pointfree funcoids f and g

1°. If $\operatorname{im} f \sqsupseteq \operatorname{dom} g$ then $\operatorname{im}(g \circ f) = \operatorname{im} g$, provided that the posets $\operatorname{Src} f$, $\operatorname{Dst} f = \operatorname{Src} g$ and $\operatorname{Dst} g$ have greatest elements and $\operatorname{Src} g$ and $\operatorname{Dst} g$ are strongly separable.

2°. If $\operatorname{im} f \sqsubseteq \operatorname{dom} g$ then $\operatorname{dom}(g \circ f) = \operatorname{dom} g$, provided that the posets $\operatorname{Dst} g$, $\operatorname{Dst} f = \operatorname{Src} g$ and $\operatorname{Src} f$ have greatest elements and $\operatorname{Dst} f$ and $\operatorname{Src} f$ are strongly separable.

PROOF.

1°. $\operatorname{im}(g \circ f) = \langle g \circ f\rangle \top^{\operatorname{Src} f} = \langle g\rangle\langle f\rangle \top^{\operatorname{Src} f} \sqsubseteq \operatorname{im} g$ by strong separability of $\operatorname{Dst} g$; $\operatorname{im}(g \circ f) = \langle g \circ f\rangle \top^{\operatorname{Src} f} = \langle g\rangle \operatorname{im} f \sqsupseteq \langle g\rangle \operatorname{dom} g = \operatorname{im} g$ by strong separability of $\operatorname{Dst} g$ and proposition 1536.

2°. $\operatorname{dom}(g \circ f) = \operatorname{im}(f^{-1} \circ g^{-1})$ what by the proved is equal to $\operatorname{im} f^{-1}$ that is $\operatorname{dom} f$.

□