$$\begin{aligned}
K\ \delta\ I' \sqcup^{\mathfrak{Z}_1} J' &\Leftrightarrow \\
(I' \sqcup^{\mathfrak{Z}_1} J') \sqcap^{\mathfrak{B}} \alpha K \neq \bot^{\mathfrak{B}} &\Leftrightarrow \\
(I' \sqcup^{\mathfrak{B}} J') \sqcap \alpha K \neq \bot^{\mathfrak{B}} &\Leftrightarrow \\
(I' \sqcap^{\mathfrak{B}} \alpha K) \sqcup (J' \sqcap^{\mathfrak{B}} \alpha K) \neq \bot^{\mathfrak{B}} &\Leftrightarrow \\
I' \sqcap^{\mathfrak{B}} \alpha K \neq \bot^{\mathfrak{B}} \vee J' \sqcap^{\mathfrak{B}} \alpha K \neq \bot^{\mathfrak{B}} &\Leftrightarrow \\
K\ \delta\ I' \vee K\ \delta\ J'&
\end{aligned}$$

and

$$\begin{aligned}
I \sqcup^{\mathfrak{Z}_0} J\ \delta\ K' &\Leftrightarrow \\
K' \sqcap^{\mathfrak{B}} \alpha(I \sqcup^{\mathfrak{Z}_0} J) \neq \bot^{\mathfrak{B}} &\Leftrightarrow \\
K' \sqcap^{\mathfrak{B}} (\alpha I \sqcup \alpha J) \neq \bot^{\mathfrak{B}} &\Leftrightarrow \\
(K' \sqcap^{\mathfrak{B}} \alpha I) \sqcup (K' \sqcap^{\mathfrak{B}} \alpha J) \neq \bot^{\mathfrak{B}} &\Leftrightarrow \\
K' \sqcap^{\mathfrak{B}} \alpha I \neq \bot^{\mathfrak{B}} \vee K' \sqcap^{\mathfrak{B}} \alpha J \neq \bot^{\mathfrak{B}} &\Leftrightarrow \\
I\ \delta\ K' \vee J\ \delta\ K'.&
\end{aligned}$$

That is the formulas (21) are true.

Accordingly the above δ can be continued to the relation $[f]$ for some $f \in \mathsf{pFCD}(\mathfrak{A}, \mathfrak{B})$.

$\forall X \in \mathfrak{Z}_0, Y \in \mathfrak{Z}_1 : (Y \sqcap^{\mathfrak{B}} \langle f\rangle X \neq \bot^{\mathfrak{B}} \Leftrightarrow X\ [f]\ Y \Leftrightarrow Y \sqcap^{\mathfrak{B}} \alpha X \neq \bot^{\mathfrak{B}})$, consequently $\forall X \in \mathfrak{Z}_0 : \alpha X = \langle f\rangle X$ because our filtrator is with separable core. So $\langle f\rangle$ is a continuation of α.

□

THEOREM 1511. Let $(\mathfrak{A}, \mathfrak{Z}_0)$ and $(\mathfrak{B}, \mathfrak{Z}_1)$ be primary filtrators over boolean lattices. If $\alpha \in \mathfrak{B}^{\mathfrak{Z}_0}$, $\beta \in \mathfrak{A}^{\mathfrak{Z}_1}$ are functions such that $Y \not\asymp \alpha X \Leftrightarrow X \not\asymp \beta Y$ for every $X \in \mathfrak{Z}_0$, $Y \in \mathfrak{Z}_1$, then there exists exactly one pointfree funcoid $f : \mathfrak{A} \to \mathfrak{B}$ such that $\langle f\rangle|_{\mathfrak{Z}_0} = \alpha$, $\langle f^{-1}\rangle|_{\mathfrak{Z}_1} = \beta$.

PROOF. Prove $\alpha(I \sqcup J) = \alpha I \sqcup \alpha J$. Really, $Y \not\asymp \alpha(I \sqcup J) \Leftrightarrow I \sqcup J \not\asymp \beta Y \Leftrightarrow I \not\asymp \beta Y \vee J \not\asymp \beta Y \Leftrightarrow Y \not\asymp \alpha I \vee Y \not\asymp \alpha J \Leftrightarrow Y \not\asymp \alpha I \sqcup \alpha J$. So $\alpha(I \sqcup J) = \alpha I \sqcup \alpha J$ by star-separability. Similarly $\beta(I \sqcup J) = \beta I \sqcup \beta J$.

Thus by the theorem above there exists a pointfree funcoid f such that $\langle f\rangle|_{\mathfrak{Z}_0} = \alpha$, $\langle f^{-1}\rangle|_{\mathfrak{Z}_1} = \beta$.

That this pointfree funcoid is unique, follows from the above. □

PROPOSITION 1512. Let $(\operatorname{Src} f, \mathfrak{Z}_0)$ be a primary filtrator over a bounded distributive lattice and $(\operatorname{Dst} f, \mathfrak{Z}_1)$ be a primary filtrator over boolean lattice. If S is a generalized filter base on $\operatorname{Src} f$ then $\langle f\rangle \bigsqcap^{\operatorname{Src} f} S = \bigsqcap^{\operatorname{Dst} f}\langle\langle f\rangle\rangle^* S$ for every pointfree funcoid f.

PROOF. First the meets $\bigsqcap^{\operatorname{Src} f} S$ and $\bigsqcap^{\operatorname{Dst} f}\langle\langle f\rangle\rangle^* S$ exist by corollary 515.

$(\operatorname{Src} f, \mathfrak{Z}_0)$ is a binarily meet-closed filtrator by corollary 533 and with separable core by theorem 534; thus we can apply theorem 1509 (up $x \neq \emptyset$ is obvious).

$\langle f\rangle \bigsqcap^{\operatorname{Src} f} S \sqsubseteq \langle f\rangle X$ for every $X \in S$ because $\operatorname{Dst} f$ is strongly separable by proposition 576 and thus $\langle f\rangle \bigsqcap^{\operatorname{Src} f} S \sqsubseteq \bigsqcap^{\operatorname{Dst} f}\langle\langle f\rangle\rangle^* S$.