1°. If $\mathfrak{A}$ is separable, for given value of $\langle f\rangle$ there exists no more than one $f\in\mathsf{pFCD}(\mathfrak{A},\mathfrak{B})$.
2°. If $\mathfrak{A}$ and $\mathfrak{B}$ are separable, for given value of $[f]$ there exists no more than one $f\in\mathsf{pFCD}(\mathfrak{A},\mathfrak{B})$.

PROOF. Let $f,g\in\mathsf{pFCD}(\mathfrak{A},\mathfrak{B})$.

1°. Let $\langle f\rangle=\langle g\rangle$. Then for every $x\in\mathfrak{A}$, $y\in\mathfrak{B}$ we have

$$x\not\asymp\langle f^{-1}\rangle y\Leftrightarrow y\not\asymp\langle f\rangle x\Leftrightarrow y\not\asymp\langle g\rangle x\Leftrightarrow x\not\asymp\langle g^{-1}\rangle y$$

and thus by separability of $\mathfrak{A}$ we have $\langle f^{-1}\rangle y=\langle g^{-1}\rangle y$ that is $\langle f^{-1}\rangle=\langle g^{-1}\rangle$ and so $f=g$.

2°. Let $[f]$=$[g]$. Then for every $x\in\mathfrak{A}$, $y\in\mathfrak{B}$ we have

$$x\not\asymp\langle f^{-1}\rangle y\Leftrightarrow x\,[f]\,y\Leftrightarrow x\,[g]\,y\Leftrightarrow x\not\asymp\langle g^{-1}\rangle y$$

and thus by separability of $\mathfrak{A}$ we have $\langle f^{-1}\rangle y=\langle g^{-1}\rangle y$ that is $\langle f^{-1}\rangle=\langle g^{-1}\rangle$. Similarly we have $\langle f\rangle=\langle g\rangle$. Thus $f=g$.

□

PROPOSITION 1496. If $\operatorname{Src} f$ and $\operatorname{Dst} f$ have least elements, then $\langle f\rangle\bot^{\operatorname{Src} f}=\bot^{\operatorname{Dst} f}$ for every pointfree funcoid f.

PROOF. $y\not\asymp\langle f\rangle\bot^{\operatorname{Src} f}\Leftrightarrow\bot^{\operatorname{Src} f}\not\asymp\langle f^{-1}\rangle y\Leftrightarrow 0$ for every $y\in\operatorname{Dst} f$. Thus $\langle f\rangle\bot^{\operatorname{Src} f}\asymp\langle f\rangle\bot^{\operatorname{Src} f}$. So $\langle f\rangle\bot^{\operatorname{Src} f}=\bot^{\operatorname{Dst} f}$. □

PROPOSITION 1497. If $\operatorname{Dst} f$ is strongly separable then $\langle f\rangle$ is a monotone function (for a pointfree funcoid f).

PROOF.

$$\begin{aligned}a\sqsubseteq b\Rightarrow&\\ \forall x\in\operatorname{Dst} f:(a\not\asymp\langle f^{-1}\rangle x\Rightarrow b\not\asymp\langle f^{-1}\rangle x)\Rightarrow&\\ \forall x\in\operatorname{Dst} f:(x\not\asymp\langle f\rangle a\Rightarrow x\not\asymp\langle f\rangle b)\Leftrightarrow&\\ \star\langle f\rangle a\subseteq\star\langle f\rangle b\Rightarrow&\\ \langle f\rangle a\sqsubseteq\langle f\rangle b.&\end{aligned}$$

□

THEOREM 1498. Let f be a pointfree funcoid from a starrish join-semilattice $\operatorname{Src} f$ to a separable starrish join-semilattice $\operatorname{Dst} f$. Then $\langle f\rangle(i\sqcup j)=\langle f\rangle i\sqcup\langle f\rangle j$ for every $i,j\in\operatorname{Src} f$.

PROOF.

$$\begin{aligned}\star\langle f\rangle(i\sqcup j)=&\\ \left\{\frac{y\in\operatorname{Dst} f}{y\not\asymp\langle f\rangle(i\sqcup j)}\right\}=&\\ \left\{\frac{y\in\operatorname{Dst} f}{i\sqcup j\not\asymp\langle f^{-1}\rangle y}\right\}=&\\ \left\{\frac{y\in\operatorname{Dst} f}{i\not\asymp\langle f^{-1}\rangle y\vee j\not\asymp\langle f^{-1}\rangle y}\right\}=&\\ \left\{\frac{y\in\operatorname{Dst} f}{y\not\asymp\langle f\rangle i\vee y\not\asymp\langle f\rangle j}\right\}=&\\ \left\{\frac{y\in\operatorname{Dst} f}{y\not\asymp\langle f\rangle i\sqcup\langle f\rangle j}\right\}=&\\ \star(\langle f\rangle i\sqcup\langle f\rangle j).&\end{aligned}$$