OBVIOUS 1473. $\operatorname{xlim}_x f = \left\{ \frac{\nu\circ f|_{\langle\mu\rangle^*@\{x\}}\circ\uparrow r}{r\in G} \right\}$.

REMARK 1474. $\operatorname{xlim}_x f$ is the same for funcoids μ and $\operatorname{Compl}\mu$.

The function τ will define an injection from the set of points of the space ν ("numbers", "points", or "vectors") to the set of all (generalized) limits (i.e. values which $\operatorname{xlim}_x f$ may take).

DEFINITION 1475. $\tau(y) \overset{\text{def}}{=} \left\{ \frac{\langle\mu\rangle^*@\{x\}\times^{\mathsf{FCD}}\langle\nu\rangle^*@\{y\}}{x\in D} \right\}$.

PROPOSITION 1476. $\tau(y) = \left\{ \frac{(\langle\mu\rangle^*@\{x\}\times^{\mathsf{FCD}}\langle\nu\rangle^*@\{y\})\circ\uparrow r}{r\in G} \right\}$ for every (fixed) $x \in D$.

PROOF.

$$\begin{aligned}
&(\langle\mu\rangle^*@\{x\}\times^{\mathsf{FCD}}\langle\nu\rangle^*@\{y\})\circ\uparrow r = \\
&\left\langle\uparrow r^{-1}\right\rangle\langle\mu\rangle^*@\{x\}\times^{\mathsf{FCD}}\langle\nu\rangle^*@\{y\} = \\
&\langle\mu\rangle\left\langle\uparrow r^{-1}\right\rangle^*@\{x\}\times^{\mathsf{FCD}}\langle\nu\rangle^*@\{y\} = \\
&\quad\langle\mu\rangle^*@\{r^{-1}x\}\times^{\mathsf{FCD}}\langle\nu\rangle^*@\{y\} \in \left\{ \frac{\langle\mu\rangle^*@\{x\}\times^{\mathsf{FCD}}\langle\nu\rangle^*@\{y\}}{x\in D} \right\}.
\end{aligned}$$

Reversely $\langle\mu\rangle^*@\{x\}\times^{\mathsf{FCD}}\langle\nu\rangle^*@\{y\} = (\langle\mu\rangle^*@\{x\}\times^{\mathsf{FCD}}\langle\nu\rangle^*@\{y\})\circ\uparrow e$ where e is the identify element of G. □

PROPOSITION 1477. $\tau(y) = \operatorname{xlim}(\langle\mu\rangle^*@\{x\}\times^{\mathsf{FCD}}\uparrow^{\operatorname{Base}(\operatorname{Ob}\nu)}\{y\})$ (for every x). Informally: Every $\tau(y)$ is a generalized limit of a constant funcoid.

PROOF.

$$\begin{aligned}
&\operatorname{xlim}(\langle\mu\rangle^*@\{x\}\times^{\mathsf{FCD}}\uparrow^{\operatorname{Base}(\operatorname{Ob}\nu)}\{y\}) = \\
&\left\{ \frac{\nu\circ(\langle\mu\rangle^*@\{x\}\times^{\mathsf{FCD}}\uparrow^{\operatorname{Base}(\operatorname{Ob}\nu)}\{y\})\circ\uparrow r}{r\in G} \right\} = \\
&\left\{ \frac{(\langle\mu\rangle^*@\{x\}\times^{\mathsf{FCD}}\langle\nu\rangle^*@\{y\})\circ\uparrow r}{r\in G} \right\} = \tau(y).
\end{aligned}$$

□

THEOREM 1478. If f is a function and $f|_{\langle\mu\rangle^*@\{x\}} \in \mathrm{C}(\mu,\nu)$ and $\langle\mu\rangle^*@\{x\} \sqsupseteq \uparrow^{\operatorname{Ob}\mu}\{x\}$ then $\operatorname{xlim}_x f = \tau(fx)$.

PROOF. $f|_{\langle\mu\rangle^*@\{x\}}\circ\mu \sqsubseteq \nu\circ f|_{\langle\mu\rangle^*@\{x\}} \sqsubseteq \nu\circ f$; thus $\langle f\rangle\langle\mu\rangle^*@\{x\} \sqsubseteq \langle\nu\rangle\langle f\rangle^*@\{x\}$; consequently we have

$$\nu \sqsupseteq \langle\nu\rangle\langle f\rangle^*@\{x\}\times^{\mathsf{FCD}}\langle\nu\rangle\langle f\rangle^*@\{x\} \sqsupseteq \langle f\rangle\langle\mu\rangle^*@\{x\}\times^{\mathsf{FCD}}\langle\nu\rangle\langle f\rangle^*@\{x\}.$$

$$\begin{aligned}
&\nu\circ f|_{\langle\mu\rangle^*@\{x\}} \sqsupseteq \\
&(\langle f\rangle\langle\mu\rangle^*@\{x\}\times^{\mathsf{FCD}}\langle\nu\rangle\langle f\rangle^*@\{x\})\circ f|_{\langle\mu\rangle^*@\{x\}} = \\
&(f|_{\langle\mu\rangle^*\{x\}})^{-1}\langle f\rangle\langle\mu\rangle^*@\{x\}\times^{\mathsf{FCD}}\langle\nu\rangle\langle f\rangle^*@\{x\} \sqsupseteq \\
&\left\langle\mathrm{id}^{\mathsf{FCD}}_{\operatorname{dom} f|_{\langle\mu\rangle^*\{x\}}}\right\rangle\langle\mu\rangle^*@\{x\}\times^{\mathsf{FCD}}\langle\nu\rangle\langle f\rangle^*@\{x\} \sqsupseteq \\
&\operatorname{dom} f|_{\langle\mu\rangle^*@\{x\}}\times^{\mathsf{FCD}}\langle\nu\rangle\langle f\rangle^*@\{x\} = \\
&\langle\mu\rangle^*@\{x\}\times^{\mathsf{FCD}}\langle\nu\rangle\langle f\rangle^*@\{x\}.
\end{aligned}$$