

Convergence of functors

18.1. Convergence

The following generalizes the well-known notion of a filter convergent to a point or to a set:

DEFINITION 1450. A filter $\mathcal{F} \in \mathcal{F}(\text{Dst } \mu)$ converges to a filter $\mathcal{A} \in \mathcal{F}(\text{Src } \mu)$ regarding a functor μ ($\mathcal{F} \xrightarrow{\mu} \mathcal{A}$) iff $\mathcal{F} \sqsubseteq \langle \mu \rangle \mathcal{A}$.

DEFINITION 1451. A functor f converges to a filter $\mathcal{A} \in \mathcal{F}(\text{Src } \mu)$ regarding a functor μ where $\text{Dst } f = \text{Dst } \mu$ (denoted $f \xrightarrow{\mu} \mathcal{A}$) iff $\text{im } f \sqsubseteq \langle \mu \rangle \mathcal{A}$ that is iff $\text{im } f \xrightarrow{\mu} \mathcal{A}$.

DEFINITION 1452. A functor f converges to a filter $\mathcal{A} \in \mathcal{F}(\text{Src } \mu)$ on a filter $\mathcal{B} \in \mathcal{F}(\text{Src } f)$ regarding a functor μ where $\text{Dst } f = \text{Dst } \mu$ iff $f|_{\mathcal{B}} \xrightarrow{\mu} \mathcal{A}$.

OBVIOUS 1453. A functor f converges to a filter $\mathcal{A} \in \mathcal{F}(\text{Src } \mu)$ on a filter $\mathcal{B} \in \mathcal{F}(\text{Src } f)$ regarding a functor μ iff $\langle f \rangle \mathcal{B} \sqsubseteq \langle \mu \rangle \mathcal{A}$.

REMARK 1454. We can define also convergence for a reloid $f: \mathcal{F} \xrightarrow{\mu} \mathcal{A} \Leftrightarrow \text{im } f \sqsubseteq \langle \mu \rangle \mathcal{A}$ or what is the same $f \xrightarrow{\mu} \mathcal{A} \Leftrightarrow (\text{FCD})f \xrightarrow{\mu} \mathcal{A}$.

THEOREM 1455. Let f, g be functors, μ, ν be endofunctors, $\text{Dst } f = \text{Src } g = \text{Ob } \mu$, $\text{Dst } g = \text{Ob } \nu$, $\mathcal{A} \in \mathcal{F}(\text{Ob } \mu)$. If $f \xrightarrow{\mu} \mathcal{A}$,

$$g|_{\langle \mu \rangle \mathcal{A}} \in C(\mu \sqcap (\langle \mu \rangle \mathcal{A} \times^{\text{FCD}} \langle \mu \rangle \mathcal{A}), \nu),$$

and $\langle \mu \rangle \mathcal{A} \sqsupseteq \mathcal{A}$, then $g \circ f \xrightarrow{\nu} \langle g \rangle \mathcal{A}$.

PROOF.

$$\begin{aligned} \text{im } f &\sqsubseteq \langle \mu \rangle \mathcal{A}; \\ \langle g \rangle \text{im } f &\sqsubseteq \langle g \rangle \langle \mu \rangle \mathcal{A}; \\ \text{im}(g \circ f) &\sqsubseteq \langle g|_{\langle \mu \rangle \mathcal{A}} \rangle \langle \mu \rangle \mathcal{A}; \\ \text{im}(g \circ f) &\sqsubseteq \langle g|_{\langle \mu \rangle \mathcal{A}} \rangle \langle \mu \sqcap (\langle \mu \rangle \mathcal{A} \times^{\text{FCD}} \langle \mu \rangle \mathcal{A}) \rangle \mathcal{A}; \\ \text{im}(g \circ f) &\sqsubseteq \langle g|_{\langle \mu \rangle \mathcal{A}} \circ (\mu \sqcap (\langle \mu \rangle \mathcal{A} \times^{\text{FCD}} \langle \mu \rangle \mathcal{A})) \rangle \mathcal{A}; \\ \text{im}(g \circ f) &\sqsubseteq \langle \nu \circ g|_{\langle \mu \rangle \mathcal{A}} \rangle \mathcal{A}; \\ \text{im}(g \circ f) &\sqsubseteq \langle \nu \circ g \rangle \mathcal{A}; \\ g \circ f &\xrightarrow{\nu} \langle g \rangle \mathcal{A}. \end{aligned}$$

□

COROLLARY 1456. Let f, g be functors, μ, ν be endofunctors, $\text{Dst } f = \text{Src } g = \text{Ob } \mu$, $\text{Dst } g = \text{Ob } \nu$, $\mathcal{A} \in \mathcal{F}(\text{Ob } \mu)$. If $f \xrightarrow{\mu} \mathcal{A}$, $g \in C(\mu, \nu)$, and $\langle \mu \rangle \mathcal{A} \sqsupseteq \mathcal{A}$ then $g \circ f \xrightarrow{\nu} \langle g \rangle \mathcal{A}$.

PROOF. From the last theorem and theorem 1184.

□