

$3^\circ \Rightarrow 4^\circ$. Suppose $\alpha \in \text{atoms}^3 \text{Cor } \Omega$. Then $\exists X \in \text{up } \Omega : \alpha \not\sqsubseteq X$. Therefore $\alpha \notin \text{atoms}^3 \text{Cor } \Omega$. So $\text{atoms}^3 \text{Cor } \Omega_{1a} = \emptyset$ and thus by atomicity $\text{Cor } \Omega_{1a} = \perp^3$.

□

COROLLARY 1437. $\text{Cor } \Omega^{\text{FCD}} = \perp$.

PROPOSITION 1438. The following is an implications tuple:

- 1°. $(\mathfrak{A}, \mathfrak{B})$ is a powerset filtrator.
- 2°. $(\mathfrak{A}, \mathfrak{B})$ is a primary filtrator over an atomic meet-semilattice with greatest element such that $\forall \alpha \in \text{atoms}^3 \exists X \in \text{coatoms}^3 : \alpha \not\sqsubseteq X$.
- 3°. \mathfrak{A} is a complete lattice, $\forall \alpha \in \text{atoms}^3 \exists X \in \text{coatoms}^3 : \alpha \not\sqsubseteq X$ and $(\mathfrak{A}; \mathfrak{B})$ is a filtered filtrator over an atomic poset.
- 4°. $\Omega_{1a} = \max \left\{ \frac{\mathcal{X} \in \mathfrak{A}}{\text{Cor } \mathcal{X} = \perp^3} \right\}$

PROOF.

$1^\circ \Rightarrow 2^\circ$. Obvious.

$2^\circ \Rightarrow 3^\circ$. Obvious.

$3^\circ \Rightarrow 4^\circ$. Due the last proposition, it is enough to show that $\text{Cor } \mathcal{X} = \perp^3 \Rightarrow \mathcal{X} \sqsubseteq \Omega_{1a}$ for every $\mathcal{X} \in \mathfrak{A}$.

Let $\text{Cor } \mathcal{X} = \perp^3$ for some $\mathcal{X} \in \mathfrak{A}$. Because of our filtrator being filtered, it's enough to show $X \in \text{up } \mathcal{X}$ for every $X \in \text{up } \Omega_{1a}$. $X = a_0 \sqcap \dots \sqcap a_n$ for a_i being coatoms of \mathfrak{B} . $a_i \sqsupseteq \mathcal{X}$ because otherwise $a_i \not\sqsupseteq \text{Cor } \mathcal{X}$. So $X \in \text{up } \mathcal{X}$.

□

PROPOSITION 1439. The following is an implications tuple:

- 1°. $(\mathfrak{A}, \mathfrak{B})$ is a powerset filtrator.
- 2°. $(\mathfrak{A}, \mathfrak{B})$ is a primary filtrator over a meet-semilattice.
- 3°. $\text{up } \Omega_{1a} = \left\{ \frac{\sqcap S}{S \in \mathcal{P}_{\text{fin}} \text{coatoms}^3} \right\}$

PROOF.

$1^\circ \Rightarrow 2^\circ$. Obvious.

$2^\circ \Rightarrow 3^\circ$. Because $\left\{ \frac{\sqcap S}{S \in \mathcal{P}_{\text{fin}} \text{coatoms}^3} \right\}$ is a filter.

□

COROLLARY 1440. $\text{up } \Omega^{\text{FCD}} = \text{up } \Omega^{\text{RLD}}$.

DEFINITION 1441. $\Omega_{1c} = \bigsqcup (\text{atoms}^{\mathfrak{A}} \setminus \mathfrak{B})$.

PROPOSITION 1442. The following is an implications tuple:

- 1°. $(\mathfrak{A}; \mathfrak{B})$ is a powerset filtrator.
- 2°. $(\mathfrak{A}; \mathfrak{B})$ is a down-aligned filtered complete lattice filtrator over an atomistic poset and $\forall \alpha \in \text{atoms}^3 \exists X \in \text{coatoms}^3 : \alpha \not\sqsubseteq X$.
- 3°. $\Omega_{1c} = \Omega_{1a}$.

PROOF.

$1^\circ \Rightarrow 2^\circ$. Obvious.

$2^\circ \Rightarrow 3^\circ$. For $x \in \text{atoms}^{\mathfrak{A}} \setminus \mathfrak{B}$ we have $\text{Cor } x = \perp$ because otherwise $\perp \neq \text{Cor } x \sqsubset x$. Thus by previous $x \sqsubseteq \Omega_{1a}$ and so $\Omega_{1c} = \bigsqcup (\text{atoms}^{\mathfrak{A}} \setminus \mathfrak{B}) \sqsubseteq \Omega_{1a}$.

If $x \in \text{atoms } \Omega_{1a}$ then $x \notin \mathfrak{B}$ because otherwise $\text{Cor } x \neq \perp$. So

$$\Omega_{1a} = \bigsqcup \text{atoms } \Omega_{1a} = \bigsqcup (\text{atoms } \Omega_{1a} \setminus \mathfrak{B}) \sqsubseteq \bigsqcup (\text{atoms}^{\mathfrak{A}} \setminus \mathfrak{B}) = \Omega_{1c}.$$

□