CHAPTER 17

Generalized cofinite filters

The following is a straightforward generalization of cofinite filter.

Definition 1431. $\Omega_{1a} = \bigsqcap^{\mathfrak{A}}_{X \in \operatorname{coatoms}^{\mathfrak{Z}}} X$; $\Omega_{1b} = \bigsqcap^{\mathfrak{A}}_{X \in \operatorname{coatoms}^{\mathfrak{A}}} X$.

Proposition 1432. The following is an implications tuple:

1°. $(\mathfrak{A}, \mathfrak{Z})$ is a powerset filtrator.
2°. $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator.
3°. $\Omega_{1a} = \Omega_{1b}$ for this filtrator.

Proof.

1°⇒2°. Obvious.
2°⇒3°. Proposition 557.

□

Proposition 1433. Let $(\mathfrak{A}, \mathfrak{Z})$ be a primary filtrator. Let $\mathfrak{Z}$ be a subset of $\mathscr{P}U$. Let it be a meet-semilattice with greatest element. Let also every non-coempty cofinite set lies in $\mathfrak{Z}$. Then

$$\partial\Omega = \left\{ \frac{Y \in \mathfrak{Z}}{\operatorname{card}\operatorname{atoms}^{\mathfrak{Z}} Y \geq \omega} \right\}. \tag{18}$$

Proof. Ω exists by corollary 515.

$Y \in \partial\Omega \Leftrightarrow Y \not\asymp^{\mathfrak{A}} \bigsqcap^{\mathfrak{A}}_{X \in \operatorname{coatoms}^{\mathfrak{Z}}} X \Leftrightarrow$ (by properties of filter bases) $\Leftrightarrow \forall S \in \mathscr{P}_{\text{fin}} \operatorname{coatoms}^{\mathfrak{Z}} : Y \not\asymp^{\mathfrak{A}} \bigsqcap^{\mathfrak{A}} S \Leftrightarrow$ (corollary 533) $\Leftrightarrow \forall S \in \mathscr{P}_{\text{fin}} \operatorname{coatoms}^{\mathfrak{Z}} : Y \not\asymp \bigsqcap S \Leftrightarrow \forall K \in \mathscr{P}_{\text{fin}} U : Y \setminus K \neq \emptyset \Leftrightarrow \operatorname{card} Y \geq \omega \Leftrightarrow \operatorname{card}\operatorname{atoms}^{\mathfrak{Z}} Y \geq \omega$. (Here $\mathscr{P}_{\text{fin}}$ denotes the set of finite subsets.) □

Corollary 1434. Formula (18) holds for both reloids and funcoids.

Proof. For reloids it's straightforward, for funcoids take that they are isomorphic to filters on lattice Γ. □

Corollary 1435. $\Omega^{\mathsf{FCD}} \neq \bot^{\mathsf{FCD}}$ (for $\mathsf{FCD}(A, B)$ where $A \times B$ is an infinite set).

Proposition 1436. The following is an implications tuple:

1°. $(\mathfrak{A}, \mathfrak{Z})$ is a powerset filtrator.
2°. $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator over an atomic ideal base and $\forall \alpha \in \operatorname{atoms}^{\mathfrak{Z}} \exists X \in \operatorname{coatoms}^{\mathfrak{Z}} : a \not\sqsubseteq X$.
3°. Ω_{1a} and $\operatorname{Cor}\Omega_{1a}$ are defined, $\forall \alpha \in \operatorname{atoms}^{\mathfrak{Z}} \exists X \in \operatorname{coatoms}^{\mathfrak{Z}} : a \not\sqsubseteq X$ and $\mathfrak{Z}$ is an atomic poset.
4°. $\operatorname{Cor}\Omega_{1a} = \bot^{\mathfrak{Z}}$.

Proof.

1°⇒2°. Obvious.
2°⇒3°. Obvious.